COMPARATIVE STRUCTURAL AND ECONOMIC EVALUATION OF ALTERNATE TRUSS DESIGNS BY MEANS OF A DIGITAL COMPUTER

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Comparative Structural and Economic Evaluation of Alternate

Truss Designs by Means of a Digital Computer

by

Thomas H. Oswald

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Abstract

The simultaneous application of structural and economic design criteria for truss-type structures is studied by means of a digital computer program. From data consisting of connectivity of members, properties of member cross sections, and member loading, an input truss design is evaluated on a basis of minimum weight of material, structural acceptability as defined by specification provisions, and complexity of joint construction. Statements of type and degree of specification infractions are produced, along with recommendations for altered design parameters to eliminate the violations. An efficiency index in the form of a weight ratio between an input member design and a theoretical optimum design, is formulated and produced as additional output. Design parameters for the optimized member are generated and made available as guides to the redesign of the member. Joint complexity is evaluated on a basis of relative member width and variety of connection requirements.

The use of the program and its output are discussed and illustrated both as a method of redesign and as a possible approach to the "direct design" of a statically indeterminate truss. Incorporation of this program and more extensive variations into the Structural Engineering System Solver (STRESS) is proposed as a means of extending the usefulness of this system as a powerful design aid.

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Without the support and encouragement of one other person, the author could not have succeeded in this task. That person, to whom this work is dedicated, is his wife Elizabeth.

Table of Contents

	Page
Background	
The design process	1
Structural specifications	3
Automatic computation	5
Objectives	8
Scope of Study	
Type of structure	9
Material	9
Structural action	10
Design criteria	10
Theory and Formulation	
Statement of the problem	13
Past work in structural optimization	15
Present effort	16
Program operation	24
Proposed Method of Employment	28
Testing and Evaluation	
Statically determinate case	30
Statically indeterminate case	32
Overall conclusions and recommendations	34

Page

Appendices

A. Structural criteria considered 37

B. Analytic routine for finding weight optimum 41

C. Documentation of computer program 49

D. Illustrative problems 62

List of references 76

List of Figures and Tables

		Page
Fig. 1	Design Limitation Curve for Compression Members	17
Fig. 2	Curve of Area vs. Radius of Gyration	20
Fig. 3	Area of Acceptable Design, Compression Members	21
Fig. 4	Area of Acceptable Design, Tension Members	22
Fig. 5	Operation of Computer Program	26
Fig. 6	Design Cycle for Indeterminate Truss	33
Fig. B-1	Idealized Built-Up Section	42
Tab. B-1	Section Formulas Used by Program	44
Fig. B-2	$A_{reqd.}$ Curve, Compression Members	45
Fig. B-3	Generalized Polynomial Function	47
Fig. D-1	Statically Determinate Test Truss	63
Fig. D-2	Statically Indeterminate Test Truss	74
Tab. D-1	Convergence of Indeterminate Design Cycle	75

Background

The design process

The design of a safe, functional, and economical civil engineering structure is an exercise in the application both of well-founded physical and mathematical principles and of judgement based on the experience and resourcefulness of the designer. Given a program of requirements for a proposed structure, such as its general function and location, expected loading and approximate upper limits on its cost, the structural designer initiates a sometimes lengthy procedure which has as its ultimate objective a design which will: 1) meet all of the functional requirements, 2) safely resist all of the anticipated loads, and 3) represent the most economical configuration satisfying (1) and (2).

In many cases the design process follows a more-or-less standard sequence of steps. Following the establishment of the general performance requirements and the decision concerning the structural form to be used (truss, rigid frame, shell, etc.), the designer must formulate a set of loading conditions representing the various separate or combined load arrangements that the structure will be required to withstand. In the case of the statically determinate structure, he may then proceed to the analysis of the selected geometric configuration and arrive at the internal forces acting in each element of the structure, after which it is possible to proportion the elements directly. However, if the structural form and geometry are such that statical indeterminacy exists, an additional step must precede the analysis. This step is the selection of a trial design to be analyzed, and is necessary since in this type of structural action, the

sharing of the load-resisting function among elements is dependent upon their relative stiffnesses. The preliminary design is then analyzed for internal forces and displacements by one of the numerous widely accepted methods of structural analysis. Based on the results of the analysis, the elements of the preliminary design can then be checked against appropriate structural acceptability criteria to determine any existing need for redesign due to the structure's exceeding one or more limits of usefulness. Depending upon the extent to which the structure or one of its elements exceeds a particular limit, that element may or may not be re-proportioned. In the event of significant changes in the design of individual elements, another cycle of analysis, either partial or complete, is carried out, followed by a second application of the design criteria to evaluate the acceptability of the design. It is here that the experience and judgement of the individual designer is called upon to insure the rapid convergence of this trial-and-error process to a final, acceptable design. In the presence of experience and good judgement, accompanied by some degree of luck, the number of cycle repetitions can be minimized or even eliminated.

Throughout the discussion thus far, no mention has been made of the inclusion of economic considerations in the design process. In the practical realm of construction and operating budgets, competitive bidding, and the continuing concern for economy of labor and materials, it can be said that economic criteria of one degree or another are being continually applied in the planning of a structure, from the original decision of whether or not to build, to the preparation of the final detail drawing. Seldom if ever can

any statement of a set of economic criteria be accurately declared complete. The relative economy of two or more alternate designs for a structure is subject to the influence of a wide range of diverse considerations, from the quoted market price of materials on the day the bids are submitted, to the long term fluctuations of wage scales for painters. It is thus seen that, rather than listing in the design process separate steps for structural and economic evaluation of a trial design, it is more accurate to regard the structural evaluation, as well as the other clearly defined "steps", as concurrent with economic evaluation.

In practice, this regard for economy in design usually consists of a conscious or unconscious effort by the designer to seek trial design elements based on: 1) minimum consumption of materials, and 2) minimization of labor and maintenance costs by avoidance of unnecessary complexity in the design. Therefore, the two major economic criteria in a structural design problem may be stated as: 1) minimum volume of materials used, and 2) simplicity of construction.

Structural specifications

The term "structural specifications" usually refers to a set of technical requirements, enumerated by some person or body of authority, which when applied to the design of a structure, will lead to a design that will safely withstand the loading to which it will be subjected. This primary function of insuring safety is sometimes accompanied by other provisions intended to insure the functionality of the structure.

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To a much smaller degree, some specifications (notably those written or revised in recent years) imply a concern for economy in design by allowing a designer a wider choice of methods and materials with which to experiment, and by establishing certain standards which are conducive to economy when widely utilized. Mainly, however, the explicit economic criteria for a design must be provided separately from the structural requirements. It then becomes the designer's task to balance one against the other as he seeks a design which is safe, functional and economical.

A structural specification is usually in the form of a particular structural quantity (average stress, slenderness ratio, etc.) which is expressed as a function of certain parameters of the design and loading situation, the exceeding of which would result in a structure that is deemed unsafe or that could not be assured of serving its intended function. The design philosophy known as "allowable stress design" defines these limiting quantities as some fraction of a level at which the structure would have reached a particular "limit of usefulness", e.g., hypothetical attainment of the yield point stress, fatigue, or instability. This fraction constitutes the factor of safety, and is intended to make allowance for the uncertainties of design, such as the possibilities of overload and of imperfection in materials and size of structural members.

Thus, to a beginner in the practice of structural design, the general nature of a structural specification is that of a limit above which his design is not allowed to fall. He therefore tends to design a particular member by the best means at his disposal, and then check the pertinent specification provisions to determine whether or not he has exceeded any of the imposed

limits. With more experience, he becomes able to arrive at a trial design which will be less likely to violate a limit, and in the event that it does, he may be able to use the nature and extent of the violation to great advantage in the redesign of the element.

The designer who simply seeks a configuration which will not exceed any limit is apparently aware of only the functional and safety requirements for his structure. If the structure is also an economical one, it is probably so by accident. Rather, a good designer seeks to arrive at a design which meets the functional and safety criteria but at the same time is neither over-complicated nor wasteful of material, labor, and maintenance.

There is one additional phase involved in what might be described as the most thorough and intelligent use of specifications. This is the exploration of alternate designs in contrast to settling upon a particular one which, when viewed alone, is structurally satisfactory and economically desirable. This process of optimization is standard practice in very extensive design projects where large amounts of money and effort will be involved, but is often not practicable on lesser projects because of the amount of time and effort required to investigate alternate designs.

Automatic computation

The time requirements for the exploration of design alternatives cease to be prohibitive with the availability of modern digital computers. An often-stated justification for the widespread adaptation of computers to problems of engineering is the resulting freedom of the engineer's mind

from the "drudgery of endless numerical calculations", which allows him to use his time and talents in a more imaginative and constructive manner. The numerous special-purpose computer programs developed during the first generation of automatic computation are now giving way to more general and easier-to-use programs, employing "problem-oriented languages".

Most notable among these programming systems developed for the field of structural engineering is the STRESS language, or STRuctural Engineering System Solver(Ref. 1). At the present time, STRESS provides a powerful tool for the rapid analysis of structures to determine the unknown forces and displacements induced by a particular system of known loads and distortions. Elimination of the analysis computations, in itself, relieves the designer of a major portion of the computational labor usually associated with the design of a structure. Most methods of structural analysis derive from certain physical and mathematical principles which are widely accepted throughout the profession, and are therefore well suited to the techniques of automatic computation.

The remaining major areas of the structural design problem, that is, the synthesis of a trial design to be analyzed and, after the analysis, the evaluation of the suitability of the trial design, are different from the analysis portion in that they require the non-quantifiable elements of engineering experience and judgement. Nevertheless, the application of these experience-bred opinions and techniques often involves the use of more numbers, equations, and calculations. It is therefore conceivable that automated routines can be devised which will allow such systems as

STRESS to go beyond the analysis of an input design, in the application of programmed structural and economic criteria to evaluate the design and indicate ways in which it can be improved.

Another recently developed use of computers is the Compatible Time-Sharing System (CTSS)[Ref. 2], under which a computer can be used simultaneously by several different operators, each with his own program and problem, and who might be in widely separated locations. A combined STRESS-CTSS capability now exists in the development stage and permits an operator to communicate directly with the computer, obtaining structural analyses of complex input designs with extremely low time-lapses (fractions of a minute).

It can thus be seen that a logical extension of a system such as STRESS would be the inclusion of a routine for the automatic evaluation of an input design, which, when used with (or without) time-sharing, would enable a designer to pursue the optimization of a structure regardless of its size or complexity, and without prohibitive time consumption.

Objectives

It is against this background of newly emerging computational methods, the appearance of new specification provisions, and the resulting need for improved techniques in utilizing the new forms in conjunction with the new methods, that this study is set. The traditional role of specifications as merely a set of limits will not be entirely discredited. Rather, a new approach to the use of these limiting quantities or expressions will be proposed, and examples will be presented in illustration of the concepts involved. Specifically, the use of specification provisions as a direct aid in improving an evaluated trial design will be investigated.

As a means of exploring the adaptation of the new ideas involving specifications to automatic computation, a model computer program will be developed and tested. The term "model program" is used to point out that the routine as formulated will of necessity be of limited scope, but will serve as a guide for the construction of future programs involving different structural types and different sets of specifications.

The program will be developed in a format which provides maximum adaptability to the STRESS language, with a view toward its future annexation as an evaluation subroutine or subprogram in that system. The actual integration with STRESS and the testing of the combined program under the Compatible Time-Sharing System are beyond the scope of the present work.

Scope of Study

Type of structure

For this study to fulfill its stated objective of providing a guide for future development, a relatively simple structural form, the plane truss, was chosen. In its ideal form, a truss subjected to some system of loads is characterised by the presence of only direct stresses in its members. The existence of so-called "secondary stresses" due to joints which, by accident or by design, are not frictionless, or which introduce eccentricity of axial load, is seldom treated in an exact quantitative manner. Rather, secondary stresses are generally regarded as an effect to be minimized by proper attention to the details of joint construction. Therefore, only the effects of direct axial stress will be considered.

It will also be considered that the loading and geometry of the truss under study are fixed. To this end, the term "alternate designs" refers to designs employing different types of cross section for corresponding members.

Material

Only steel members will be considered, for the following reasons:

1) greater availability of information concerning the stress-strain behavior of steels
2) greater ease of idealizing the stress-strain characteristics and of reducing these characteristics to analytic expressions
3) restriction of STRESS to structures of linear-elastic materials.

4) infrequent occurence in practice of trusses consisting of more than one material.

Structural action

There is no limitation on the type of structural action. Statical determinacy of a truss makes possible the "direct design" of its members on a theoretical basis. But comparison of alternate designs is still a reality when a choice must be made from sets of available cross sections which approximate ideal areas and rigidities to varying degrees.

The most significant use of this study can be made when the truss in question is statically indeterminate. It is with this type of structural action that the need for evaluation and redesign is inherent, and that indications toward improvement, based on design criteria, can be most valuable.

Design criteria

As with the choice of structural type, a set of design criteria which were relatively simple was sought. A survey was made of the current editions of the three major specifications governing the design of steel structures (Refs. 3, 4, and 5), and those specification provisions which pertain to the design of truss members were abstracted and compared. The provisions taken from the American Institute of Steel Construction's Specification for the Design, Fabrication, and Erection of Structural Steel for Buildings, 1961 revision, were adopted for this study. These provisions encompass three "limits of usefulness" which can be realistically applied

to truss member design. These are:

1) average axial stress in relation to the yield-point stress
2) gross buckling of compression members
3) local buckling of projecting plate elements of compression members, based on attainment of the yield stress.

In addition to the greater adaptability of the AISC specification provisions to a study of this type, it is felt that, particularly in the treatment of axially loaded columns, the AISC equations, based on the tangent modulus theory of column strength, offer a much more rational philosophy for design than do the secant formulas of the AREA and AASHO. A discussion of the structural design criteria used in this study is found in Appendix A.

As was stated previously, the two primary economic criteria in structural design are minimum consumption of materials and simplicity of construction. In the design of trusses, minimum-weight proportioning is often the sole economic criterion applied. This approach is quite valid in theory, where the end result likely is a sketch of the truss and a tabulation of the volume of steel required. In actuality however, the final in-place cost of a truss depends upon many additional factors, such as the complexity of built-up sections, difficulty of joint construction, transportation costs, maintenance requirements, etc.

This study will consider two of these economic criteria which are most significant at the stage of design where the alternatives consist of substituting different cross sections for a particular member whose design requires modification. These are:

1) minimum weight of the principal elements, i.e. excluding rivets, fillers, gusset plates and lacing bars.

2) simplicity of joint construction.

Theory and Formulation

Statement of problem

The primary objective in the design of a steel truss is the selection of a particular type and size of cross section for each of its members. This selection is mainly dependent upon the length and loading of the member; proportioning of connections, in turn, is dependent upon the configurations arrived at for the groups of members intersecting at the various joints. The strength of connections is likewise related to the magnitude of the forces acting in the associated members.

As previously stated, the geometry of the truss being designed is assumed constant, that is, the only variations in design to be considered are the alternate choices of cross section for each member. The gross forces acting in each bar, i.e. the axial load, is determined by the analysis of the structure, whether it is done by manual computation or by computer. In this development, the determination of all gross forces will be assumed to have been by use of the STRESS system, on an IBM 7094 computer. (Future versions of STRESS will be adapted to smaller computers, such as the IBM 1620.) The problem therefore reduces to the selection of a cross section for each truss member, given its length and the magnitude and sense of its axial load. Truss members will be assumed to have ideally hinged joints, a necessary condition for the occurrence of axial forces alone. Therefore the "effective lengths" required for buckling considerations are taken as the distances from center to center of the respective terminal joints.

Application of the structural design criteria for average direct stress, gross buckling and local buckling requires the manipulation of three basic

parameters: cross sectional area for elongational rigidity, least radius of gyration for gross flexural rigidity, and thickness of projecting plate elements for local buckling rigidity. If the structural cross sections in common use consisted of solid circular or polygonal shapes with no reentrant angles, the parameters mentioned above would behave in a fairly predictable and controllable manner. Indeed one of them, the thickness of projecting plate elements would be eliminated completely. However, this is found in practice only in circular or rectangular bars, with a wide variety of angles, channels, I- and T- shaped sections and all their combinations comprising the majority of available shapes.

Several general rules of thumb become readily apparent when the structural criteria are examined. For instance it is seen that for a member of given length subjected to a given axial load, a reduction in weight should result from the use of higher strength steels, with the attendant higher allowable stresses. Also evident is the fact that, for a given grade of steel, higher allowable stresses occur with lower slenderness ratios, i.e. higher radii of gyration. Attempts to proportion a member based on satisfying the design criteria one at a time often are self-defeating. For example, if an area is selected based only on allowable average stress, and the gross buckling criterion then applied, it may be found that the slenderness ratio is too great. A further attempt to hold the area constant and increase the radius of gyration will possibly result in the attenuation of plate elements to thicknesses low enough to permit local buckling to occur. Finally, if dimensions for a particular shape could be established such that no design limitation is exceeded, the likelihood of finding that shape available in those

exact dimensions is extremely small.

For some types of load-length combinations, it may be desirable to employ a section built up of several rolled sections connected by plates or lacing bars. Such a choice might be dictated by the need for a lower slenderness ratio but with the area unchanged. Or the required area might not be attainable with any single rolled section. It is the choice between single rolled shapes and built-up sections that requires the greatest attention to the interaction between structural and economic factors. As a rule, built-up sections are less economical than rolled shapes of equal area and radius of gyration, because of the extra cost of materials, fabrication labor, and maintenance involved. In addition, latticed columns tend to have lower buckling strengths than single shapes of equal area and radius of gyration, due to the shear effect which must be considered, even in the case of axial loading.

These are only a few of the many interacting considerations which are present in the selection of a cross section for a truss member. The designer is confronted with a set of alternate manipulations of the design parameters area, radius of gyration, and thickness. He must handle them in such a manner as to satisfy the structural criteria imposed by the specifications, but at the same time prevent his solution from being unconservative of material and fabrication. The pursuit of a solution which is at the same time acceptable structurally and most desirable economically is often called the optimization of the member or structure.

Past work in structural optimization

"Structural optimization" is a term susceptible to many definitions, all

involving the same general concept of seeking the "best" solution to a problem, but all of which differ in terms of the number and nature of criteria stated or implied. A great deal of work has been done in the study of structural optimization, particularly through the use of computers.

de Neufville (Ref. 7) presents an automated routine for the trial design, analysis, and redesign of a complete steel building frame, in which he maintains that the optimization attempted is approached fairly closely for the structure as a whole, but that individual member proportions arrived at may prove unacceptable. Gray (Ref. 8) outlines a minimum-weight machine solution which considers the structural design criteria separately and then selects the design satisfying all three as the weight optimum. A mathematically elegant routine for the weight optimization of trusses, in which a trial design is optimized with respect to a concave constraint surface constructed of the various design parameter limitations, is described by Schmidt (Ref. 9).

Present effort

The concepts and procedures herein formulated constitute an extension of the lines of thought of the works mentioned above, to an application compatible with the advanced computer techniques and revised design criteria which have emerged during the last several years. Notable similarities to and differences from these previous works are:

1) Optimization efforts are centered first on individual members, with the "worth indicators" for the entire truss emerging as secondary output.

2) The three structural criteria are combined with a minimum-

weight consideration into a single analytical routine for finding the optimum design, rather than considered separately.

3) Member-centered optimization reduces Schmidt's multi-dimensional constraint surface to a two-dimensional constraint curve.

4) The present work deals with a more recent set of structural criteria. (This in itself implies no particular superiority over the past works, since these later specification requirements are equally subject to revision.)

This formulation centers around the existence of a "design parameter area" (after Schmidt) which is shown graphically for a compression member in Figure 1 as a plot of cross sectional area (A) vs. radius of gyration (R).

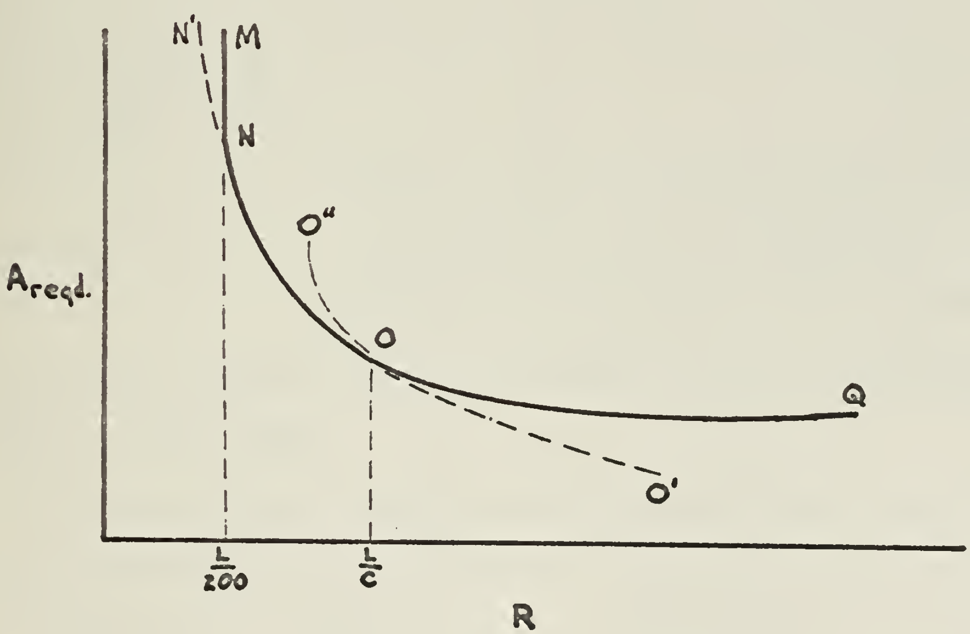


Fig. 1 Design limitation curve for compression members.

This area is divided into regions of acceptable and unacceptable design by a continuous curve MNOQ, defined by the combined criteria for average stress and gross buckling. Any point on the graph can be located by its (A, R) coordinates and therefore represents a "design point", or some section having those values of cross sectional area and radius of gyration. Whether or not a design violates one or both of the criteria is determined by the location of its design point with respect to the curve. It should be noted that R represents the least radius of gyration for sections which are not equally rigid about both principal axes of inertia. It is also pointed out that the axial force P and the length L are given for the particular member and analysis cycle under consideration. The A and R scales are defined by

$$A = \frac{P}{F_a}$$

$$R = \frac{L}{\frac{L}{R}}$$

where F_a is the allowable stress as given by the specification (see Appendix A.) and $\frac{L}{R}$ is the slenderness ratio of the member.

Any consistent system of units may be used, but the inches-kips system will be adopted in this study.

It can be seen that a generalized pair of coordinate axes, having $0 \leq A \leq \infty$ and $0 \leq r \leq \infty$ can be used for any member, the relative positions of the constraint curve MNOQ being determined solely by the values of P and L for each member. The three segments of the constraint curve for compression members are defined as follows:

1) Segment MN represents a constant value of R, given by

$$R = \frac{L}{200}$$

which is the minimum value of R allowed for a given value of L.

2) Segments NO and OQ are respectively the hyperbolic and cubic curves of required area for full allowable stress, given P and varying $\frac{L}{R}$, as given by the allowable stress formulas of Appendix A.

3) NN' and OO' are extensions of the Euler-hyperbola segment NO, and OO'' is the extension of the cubic curve OQ. NO and OQ are tangent at O.

The constraint curve for a tension member is similar to that for compression, except that the vertical occurs at $R = \frac{L}{240}$, and segment NOQ is replaced by a line of constant area at $A = \frac{P}{0.6 \times Fy}$. These curves represent the "required area" for attainment of full allowable stress, for varying values of R.

The third structural criterion, prevention of local buckling, is associated with the third design parameter, thickness of projecting elements. Expressions are derived in Appendix B for 15 shapes, of the form

$$A = f(K, W, R^2)$$

where K is a function of the physical proportions of the section

and W is the maximum allowable width-thickness ratio under the local-buckling specification.

The resulting parabola, giving "furnished area" as a function of "furnished radius of gyration" is illustrated in Figure 2.

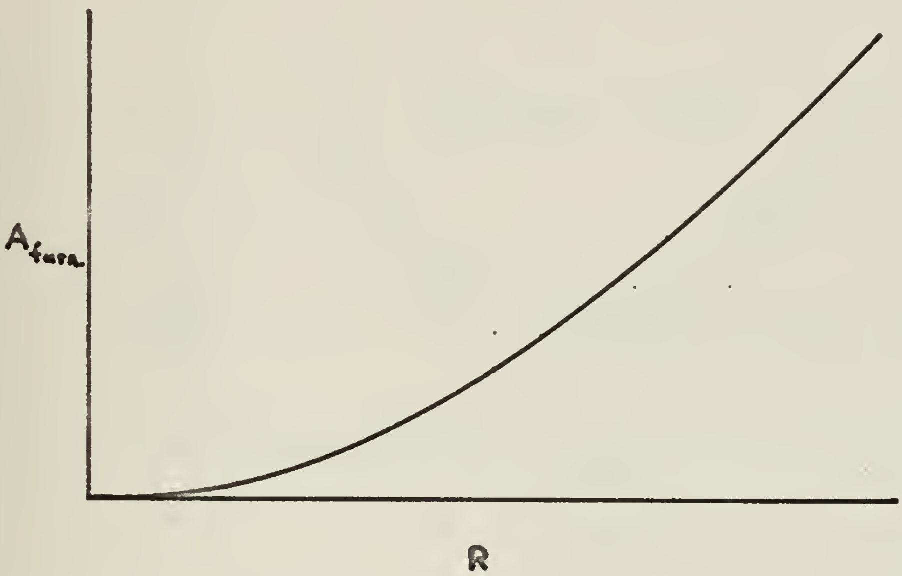


Fig. 2 Curve of area vs. radius of gyration.

These curves likewise define areas of acceptable and unacceptable design, but only with respect to local buckling. Points lying below the parabola represent proportions for which projecting elements would be dangerously thin. Note that this curve is independent of the load and length of the member.

Having defined these two curves and their associated areas of acceptable and unacceptable design points, we now proceed to their employment in the optimization process. In order to consider the three structural and one economic design criteria simultaneously, the two curves of A vs. R are superimposed on a common set of axes, resulting in the curve of Figure 3 (compression members only).

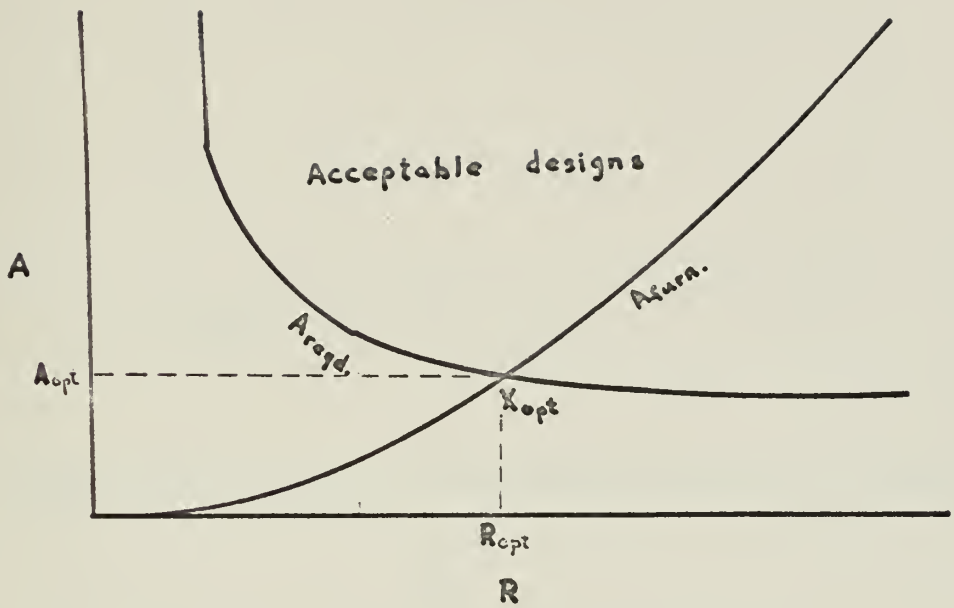


Fig. 3 Area of acceptable design, compression members.

At this point it is seen that the region of structurally acceptable design lies above and between the $A_{furn.}$ and $A_{reqd.}$ curves. Within this region the one point of minimum area (and therefore minimum weight) is $X_{opt.}$, the point of intersection of the two curves. The coordinates of this point are taken as the area and radius of gyration of a section of the same type (angle, channel, etc.) and same geometric proportions (D/B, etc.) as that of the trial design member, and which further possesses minimum area, minimum allowable thickness, and maximum allowable stress. Since tension members are not susceptible to local buckling, the parabolic section curve has no meaning when superimposed on a design parameter graph for a member in tension. Therefore, the optimum area for a tension member is taken as the area required to produce exactly the allowable tensile stress, and the optimum radius of gyration as that for which the slenderness ratio is the maximum permissible.

This point is shown in Figure 4.

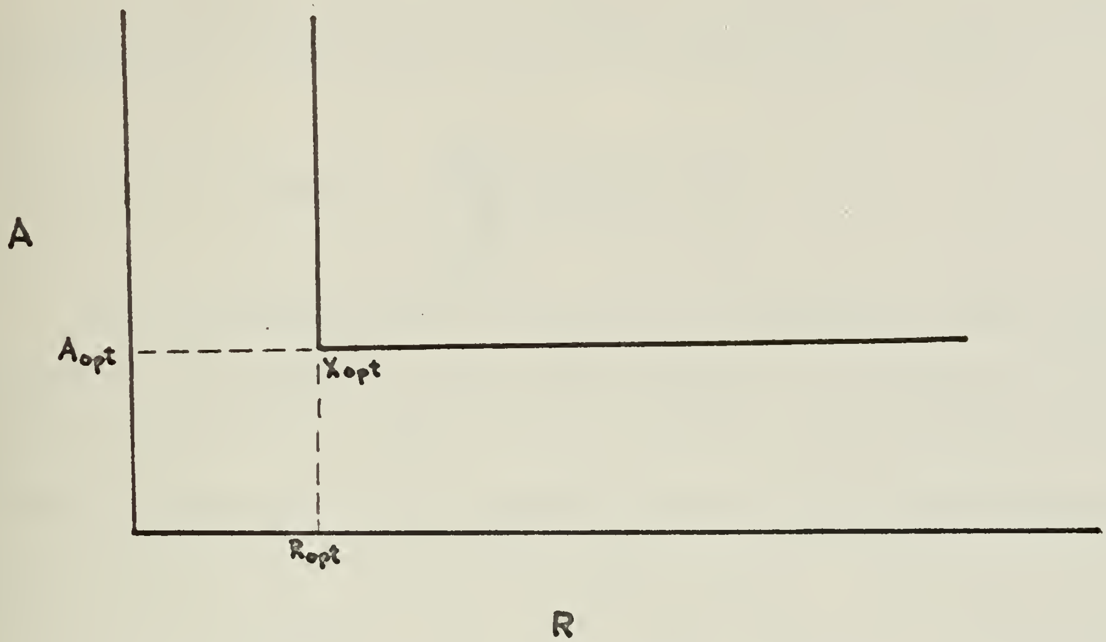


Fig. 4 Area of acceptable design, tension members.

With the determination of this theoretical optimum section, it is possible next to compare the area of the actual trial section with the area of the optimum section and thereby arrive at a numerical indicator expressing the relative underweight or overweight of the trial member. This indicator is given the name "weight merit factor" and is computed by

$$WMF = \frac{A_{trial}}{A_{opt.}}$$

Note that a WMF < 1.0 definitely indicates that the trial section violates one, two, or all three of the structural criteria, whereas a WMF > 1.0 may or may not define an overweight but structurally satisfactory member, depending upon the radius of gyration of the trial design. The analytical methods derived for the location of the intersection point by the computer are described in Appendix B..

In order to provide a similar index by which altered truss designs can be compared, a weight merit factor for the entire truss (WMFT) is computed as

$$WMFT = \frac{\sum_{mbrs.} (A_{trial} \times L)}{\sum_{mbrs.} (A_{opt.} \times L)}$$

The remaining economic criterion to be considered, that of joint complexity, is handled in a much simpler manner. The input data to the computer and the way in which this information is stored enables the machine to keep a running count of the number of joints having intersecting members which require so-called double-plane gusset plates. An example of a "double plane member" would be a box section, which requires two gusset plates separated by the width of the member transverse to the plane of the truss. Also computed is the ratio of the width of the widest such member to the narrowest at each joint, as a measure of the relative difficulty of fit among adjacent members. The tabulation of numbers of joints which require all single, all double, and mixed gusset connections, and the tabulation of member width ratios for double-gusset joints, provide the designer with a second numerical index which he may use in the comparison of alternate designs. The joint complexity tabulations are especially useful when the truss is made up of a relatively large number of members.

Other indicators, such as a tabulation of number of members requiring lacing, number of different shapes employed throughout the truss, etc., could be devised and programmed; but in most cases, these considerations are self-evident to the engineer as he works with the design and are hardly worth inclusion in the computer routine.

The operation of the computer program is outlined in the next section, and the program itself is more fully described in Appendix C.

Program operation

The computer program is designed to evaluate a particular truss design, the properties of which are its input data. As was stated in the section on Objectives, the program is intended to serve as a model for some future evaluation subroutine to be included in the STRESS program. Because of this, the quantities required for input to the evaluation program were selected and given the same or similar nomenclature as that which they bear in the present versions of STRESS, to facilitate integration into the larger system.

Whereas STRESS presently requires only the elongational rigidity quantity, or cross-sectional area (A1) of a truss member, the evaluation program must receive additional information as to the geometrical shape and dimensions of the section, as well as the material of which the member is to be made. With this data it can compute the various quantities involved in checking a member against the structural specification provisions, as well as construct the various curves of the optimization routine. A complete discussion of the input data is given in Appendix C.

Having received the input data for a particular design, consisting of the location and properties of all the separate members, the program performs three major operations:

1) The connectivity matrix is established, which records the identification number and connection characteristics (single-

plane or double-plane) of each member intersecting at each joint. From this array, the tabulation of the previously described joint complexity information is formulated and printed as output.

2) Based on the input section properties, each member in turn is checked against the programmed structural criteria. Where some specification limit is exceeded, a violation message is printed, giving the nature and degree of the excess, plus the value of the section property which, if present, would eliminate that particular violation.

3) For each member evaluated, except those found to have zero axial force, the coordinates of the "optimum design point" (area and radius of gyration on the design parameter graph) are computed and printed, along with the weight merit factors for each individual member and the truss as a whole. A summary tabulation of the results of the specification and economy checks is then provided.

A macro-flow-chart of these major operations is given in Figure 5, which follows.

Figure 5

Operation of Computer Program

Read data on size of truss and method of connection.

↓

Read connectivity data for each member and store in connectivity matrix.

↓

Count the number of joints requiring all single- and all double-plane connections.

↓

Read section properties for one member.

↓

Compute width of member transverse to plane of truss and store for each related joint.

↓

Compute maximum slenderness ratio for member.

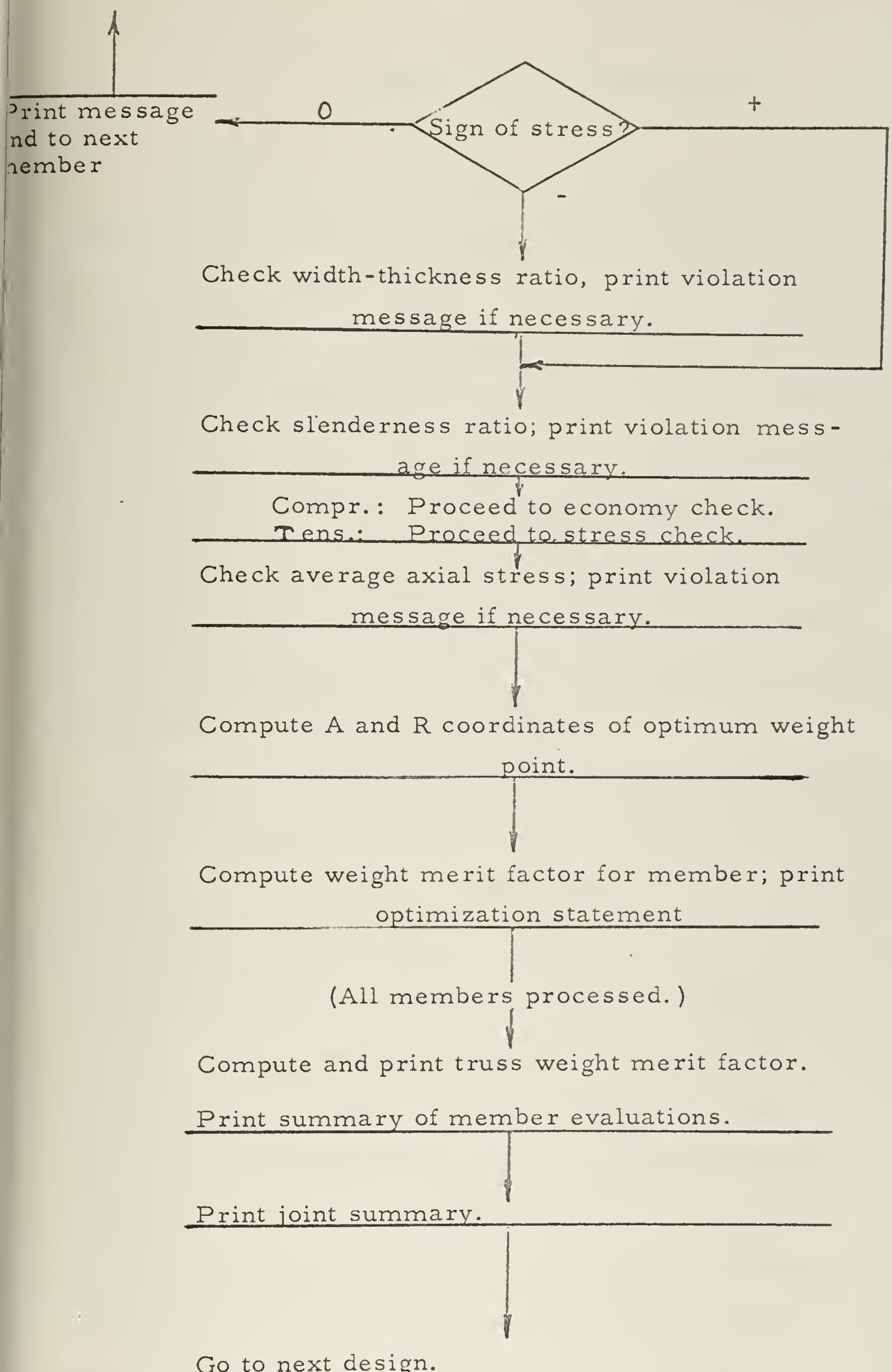
↓

Compute allowable width-thickness ratio for member.

↓

Compute average axial stress

↓


Print message
nd to next
nember
0
Sign of stress?
+
-
Check width-thickness ratio, print violation message if necessary.
Check slenderness ratio; print violation message if necessary.
Compr.: Proceed to economy check.
Tens.: Proceed to stress check.
Check average axial stress; print violation message if necessary.
Compute A and R coordinates of optimum weight point.
Compute weight merit factor for member; print optimization statement
(All members processed.)
Compute and print truss weight merit factor.
Print summary of member evaluations.
Print joint summary.
Go to next design.

Proposed Method of Employment

The effectiveness of a computer program which evaluates an input design, as a design aid, is largely determined by the skill and knowledge of the engineer who is using it. An experienced designer can better understand the significance of the numerical quantities produced, and therefore can make more intelligent use of them in the improvement of his design. Yet the usefulness of such a program is certainly not restricted to accomplished designers. The beginning student can, by making frequent use of such design aids begin to acquire an understanding of structural action and of the consequences which result from making changes in a design, without having to burden himself with a mass of computations which, in themselves, serve no useful purpose.

The employment of the concepts herein developed can be divided into two major categories paralleling the two principal structural behavior modes, statically determinate and statically indeterminate. The well-known characteristic of a determinate structure, i.e., force distribution among its members dependent only upon its geometry, make possible a "direct design" process, wherein once the analysis has produced the magnitude of the gross forces acting, the members can be proportioned once and for all, with no further analysis required. However, the member shapes and sizes first selected may or may not represent the most economical configurations. It is also possible that the first selection may violate one or more of the structural criteria, without this being readily apparent to the designer. Such a possibility is enlarged in the case of compression members, for which three parameters, area, radius of gyration, and width-thickness ratio, all

of which are complexly interrelated for structural shapes, must be manipulated to produce the "best" design. It is therefore considered that such a program as this can indeed serve a useful purpose in the design of determinate structures, both in the structural criteria check and in the determination of the efficiency of the design relative to an ideal structure.

In the second area of application, that of statically indeterminate trusses, a slightly different use is foreseen. The same functions as performed in the determinate case can ultimately be realized for an indeterminate structure which is made up of members with actual, available, non-idealized cross sections. In addition to these practical applications, the possibility exists for use of this type of routine in arriving at an initial design which will more closely resemble the final, accepted design, and which will minimize the number of analysis-evaluation-redesign cycles required for the determination of this design.

Testing and Evaluation

Statically determinate case

The testing of the concepts herein described was carried out for the statically determinate case by running a series of sample problems with successive revisions of the program. Mathematical errors and logical imperfections were corrected as they arose. It was found that the results obtained were consistent with manually prepared, graphical solutions of the test problems, in which the two curves $A_{furn.}$ and $A_{reqd.}$ were superimposed and the intersection coordinates read directly.

It is concluded that the output information concerning the nature and degree of specification violations is mathematically valid, and in the format in which it is presented it provides an easy-to-read and practical summary of the structural acceptability of the member. The provision of the quantity AREQD ("area required for this load to produce full allowable stress") is of limited usefulness for compression members, since the method of computation assumes a constant value of L/R, which is to say, constant R. Adjustment of the area of a given structural cross section without appreciably altering the radius of gyration is very difficult, if not impossible in most cases. This apparent deficiency in usefulness of the AREQD quantity is more pronounced in the regions of low R, where the allowable stress for a member of given load and length varies more rapidly with R than in the high-R range, where the $A_{reqd.}$ curve is flatter. Nevertheless, the availability of this quantity is of some value in helping the designer to sketch the relative positions of the various input-output design points if he so desires, in order to gain a clearer understanding of the

requirements for redesign of a member. The quantity AREQD for overstressed tension members, which is independent of L/R, as well as the quantity RREQD ("required radius of gyration for this length, to produce the maximum allowable slenderness ratio") for both tension and compression members are straightforward in concept and serve their intended purpose in a satisfactory manner. Interpretation of either of the quantities AREQD or RREQD independently of considering the state of the other design parameter, whether it be adequate or deficient, can lead to a second design which may violate another specification limit.

For both tension and compression members, the provision of output data for the optimum point on the design parameter graph is considered more useful in evaluation and subsequent redesign of unaccaptable or uneconomical members, since they are computed as functions of all the pertinent parameters. The quantities AOPT and ROPT are very effective in indicating to the designer a direction and distance on the graph, which he may use to great advangage in re-designing the member. Because of the V-shaped area of structurally acceptable design points produced by the two curves for compression members, it is usually desirable to redesign by first seeking a value of R as near ROPT as possible, and then a value of A not less than AOPT. By doing this, the chance of falling below either curve in the redesign is lessened.

The output quantity WMF (weight merit factor) is considered to be an excellent index of the efficiency of the cross section when interpreted concurrently with the structural acceptability evaluation. If successive design alterations produce no specification infractions and WMF's greater

than 1.0, then the quantity is perfectly valid as a measure of their relative levels of efficiency.

The summary of joint complexity of the input design is of greatest value in a truss composed of a large number of members, since it is primarily a bookkeeping routine. However, a brief perusal of the member width ratios for double-gusset joints could be of considerable value when comparing one truss design with another.

A set of input/output data for a sample statically determinate truss is given in Appendix D. In this illustrative problem, one cycle of redesign was undertaken to demonstrate the use of the evaluation output quantities.

Statically indeterminate case

Testing of the program in the design of indeterminate trusses was limited by time. However, a series of three redesign cycles was performed on a small indeterminate truss, producing results which cannot be termed conclusive on the basis of this one experiment, but which nevertheless indicate the possibility of using this program in conjunction with the STRESS analyzer to optimize an indeterminate design in comparatively few cycles.

The truss and input/output data for the analysis and evaluation cycles are presented in Appendix D. The initial design for input to the analyzer was roughly based on the level of axial force which might exist in the structure, were its indeterminacy eliminated by the removal of its redundant bar. Axial forces generated by STRESS were then input to the evaluation program along with the same member configurations input to the analyzer. In turn, the AOPT and ROPT quantities generated by the evaluator were re-

cycled into STRESS as the second trial design. Figure 6 shows schematically the procedure followed.

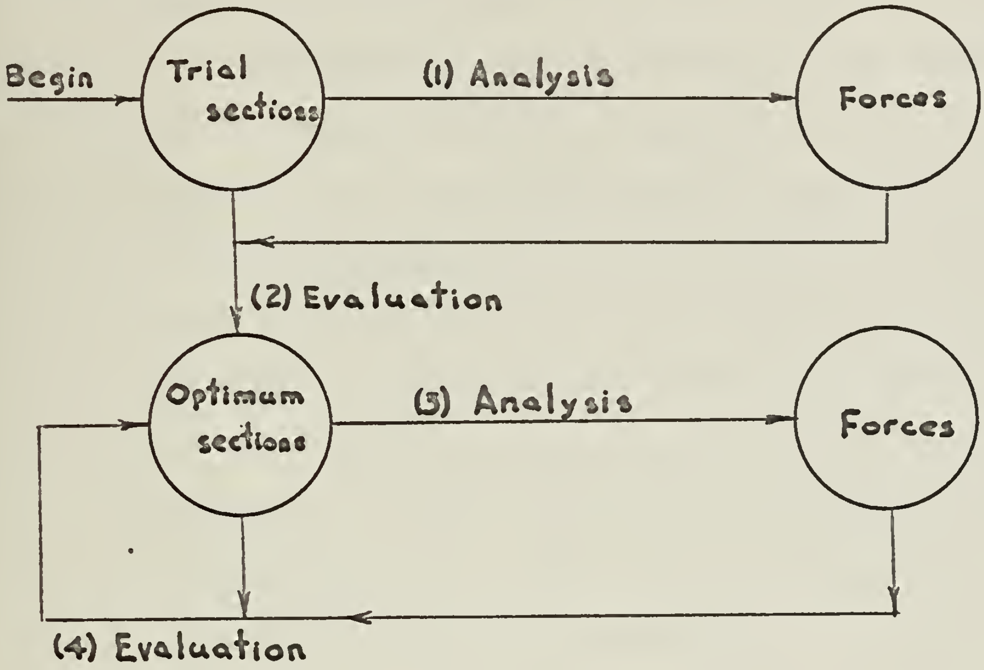


Fig. 6 Design cycle for indeterminate truss.

The convergence of the weight merit factors over three cycles indicates that a design consisting of members with parameters approximately equal to these optimum quantities would closely approach the final, accepted configuration. Coupled to STRESS so that the feedback of data from one program to the other would be automatic, this routine could perform many cycles of redesign and re-analysis, until arbitrarily established convergence criteria were met. The total time spent in these design cycles would be very short indeed, in comparison with that required for identical manual computations. The possibility of oscillation or divergence is acknowledged, but discounted, on the basis of this single test.

Overall conclusions and recommendations

In addition to the conclusions stated above, the following general evaluation of the methods that have been developed is presented:

1) While the program's function is satisfactory in the determination of optimum area and radius of gyration within the same type and overall proportions of the cross section, the case often arises where the input section is the smallest available one of that type, but may still have a WMF much greater than 1.0. It is therefore felt that a generalization of the section equations to embrace a variety of shapes would be greatly desirable. Recommendations for a different profile might be provided as well as the information now generated.

2) The expansion of the evaluation procedure to cover more sets of structural criteria, perhaps as separate subroutines, would greatly enhance the generality of the program as a design tool. Also in need of further development is the area of economic criteria. Consideration of additional economic parameters, and the possible development of a numerical index similar to WMF would also prove beneficial.

3) Certainly open to further development is the application of the general techniques of this work to other types of structures, in which the structural action is more complex and the design limitations more numerous, such as rigid frames.

4) The programming of a routine of this nature to search stored tables of section properties in the redesign phase would

eliminate another of the areas of manual involvement in intermediate procedures.

5) It is felt that the greatest opportunity for further development in this field of computer application lies in exploring the field of statically indeterminate design by procedures such as that illustrated in Appendix D.

Appendices

A. Structural Criteria Considered

In arriving at a set of structural design criteria to be included in this developmental study, it was required that the criteria selected satisfy two conditions, simplicity and rationality. A survey was made of the three major specifications governing the design of steel structures (Refs. 3, 4, and 5), and those provisions of each which pertain to the design of truss members subjected only to axial loading were compared as to their conceptual basis and the ease with which they could be programmed and linked to the quantities and techniques of the STRESS system.

It was concluded that the design criteria set forth by the 1961 A.I.S.C. Specification were most suitable for this study. A full discussion of the theoretical and empirical considerations behind the provisions is not here pertinent; such can be found in Reference 13. Instead, a brief summary of the provisions adopted, along with their general conceptual basis, will be given.

Compression members

a. The slenderness ratio (L/R) corresponding to an average stress at the "Euler buckling load" equal to 1/2 of the yield stress is given by

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

where E is the modulus of elasticity,

F_y is the yield stress

b. AISC Formula (1) then prescribes the maximum allowable axial stress for compression members having $L/R \leq C_c$, i.e. columns whose

mode of failure will be inelastic buckling, as

$$F_a = \frac{\left[1 - \frac{(L/R)^2}{2\,C_c^{\,2}}\right] F_y}{(\text{f. of s.})} \quad \text{ksi}$$

where the factor of safety

$$\text{f. of s.} = 5/3 + 3/8\left(\frac{L/R}{C_c}\right) - \frac{1}{8}\left(\frac{L/R}{C_c}\right)^3$$

c. AISC Formula (2) pertains to columns whose failure is by elastic buckling.

For $(L/R) > C_c$

$$F_a = \frac{149,000}{(L/R)^2} \quad \text{ksi}$$

d. Basis: tangent modulus theory of column strength. Formula (1) - Numerator is the Column Research Council's expression for the ultimate strength of a column. The factor of safety varies from 1.67 for $L/R \doteq 0$, to 1.92 for long columns. (Note that 1.67 is equal to the factor of safety for tension members.) Formula (2) - Allowable stress is the Euler stress divided by a f. of s. of 1.92. No distinction is made in this study between main members and secondary, or bracing members, hence AISC Formula (3) is not included.

Maximum allowable slenderness ratio:200.

Local buckling of plate elements of compression members

a. The AISC provisions on local buckling are in the form of maximum allowable width-thickness (B/T) ratios of the form

$$(\frac{B}{T})_{max.} = \frac{k}{\sqrt{F_y}}$$

where k is a specified constant depending upon the shape (tee, angle, etc.)

Rather than program these shape-dependent equations directly, it was elected to program the more general expression from which they were derived. This expression of plate buckling strength, attributable to Bryan, is

$$f_{cr} = S \frac{\pi^2 E}{12(1-\nu^2)} (\frac{t}{b})^2$$

where E is Young's modulus,

ν is Poisson's ratio,

taken as 0.3 for steel,

and K is a constant depending upon the profile of the section.

Setting $f_{cr} = f_y$ and incorporating a safety factor of approx. 1.43 (AISC's choice), the expression becomes the equation as programmed:

$$(B/T)_{max.} = 113.8 \sqrt{\frac{S}{f_y}}$$

where f_y is given in ksi,

and S = 0.43 for angles and channels,

0.70 for I and WF sections

1.28 for T sections.

Tension members

a. AISC Section 1.5.1.1 gives as the maximum allowable tensile stress, computed on the basis of net section,

$$F_t = 0.60 F_y$$

b. Maximum allowable slenderness ratio is 240 for all tension members.

c. Basis: factor of safety of 1.67 against yielding of the cross section.

B. Analytic Routine for Finding Weight Optimum

The location on the design parameter graph of the area and radius of gyration of the optimum weight cross section for compression members reduces to a routine for the simultaneous solution of two equations, one representing the area furnished, and the other representing the area required, both of which can be expressed as a function of the radius of gyration. The $A_{furn.}$ curve is a parabola symmetric about the (positive) A-axis, and with its vertex at the origin. The $A_{reqd.}$ curve, as previously illustrated, consists of three segments which are linear, hyperbolic and cubic in order of increasing R. The derivation of the expression represented by the $A_{furn.}$ parabola, and the methods programmed for finding the intersection point, will now be given.

Derivation of parabolic equation for $A_{furn.}$

The only design criterion included in the $A_{furn.}$ vs. R curve is the maximum allowable width-thickness ratio for the section. The other two limited quantities, A and R, are related by the $A_{reqd.}$ vs. R curves. Parabolic expressions of the furnished area of idealized sections as a function of R were developed and programmed for 15 cross-section configurations commonly used for truss members. Each equation includes in its derivation the thickness of plate elements equal to the element's width divided by the maximum allowable B/T ratio, i.e., the minimum allowable thickness in terms of the width. Other considerations in the development of these equations are:

1) Assumption of a weak-axis radius of gyration equal to some fraction of the overall width of the section. A list of these approximations is found in Table 6-4 of Reference 6.

2) Assumption of a constant ratio of web thickness to flange thickness, based on averages determined by sampling the AISC Steel Construction Manual tables of cross sections. These constants are as follows:

    a) Channels — 1.0

    b) WF and T sections — 0.6

    c) I-sections — 1.2 or 0.7, depending upon the weight per foot of the cross section.

3) Neglecting of "second order" areas equal to the square of the thickness, which occur at corners.

The derivation for one section will be illustrated, and the results (only) for all 15 sections will be tabulated in Table B-1.

Sample derivation

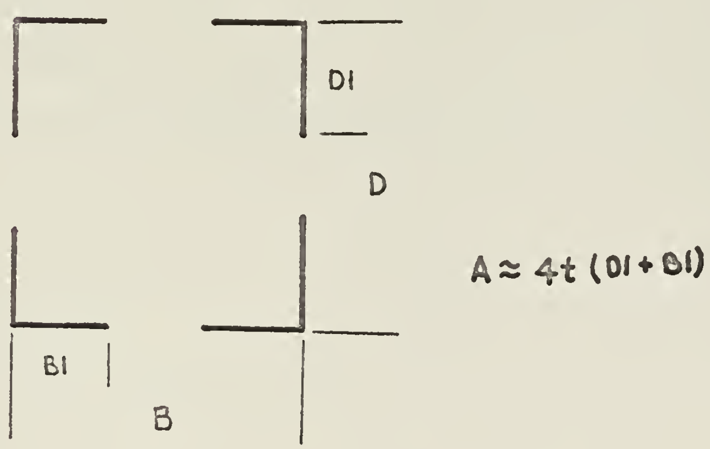


Fig. B-1 Idealized built-up section.

In the sketch above the section is built up of 4 unequal-legged angles connected in some manner such as lacing bars, perforated plates, etc. Profile proportions are

$$S = D/B$$

$$T = D1/D \quad \text{or} \quad D1 = DT = SBT$$

$$U = B1/B \quad \text{or} \quad B1 = BU$$

In the case of members built up of only two sub-elements, either T or U is 1.0; single-element members have both T and U = 1.0. For these idealizations, it is also assumed that B is the overall dimension of the section transverse to the axis of weak bending resistance. Substituting

$$R_y \qquad 0.40\ B \text{ or } B = 2.5\ R_y$$

$$\text{and} \quad t_{min.} = \frac{D1}{W} = \frac{DT}{W}$$

where W = (B/t) allowable,

into the expression for area given in Figure B-1, we get;

$$\begin{aligned} A &= 4t\,(B1 + D1) \\ &= 4Bt\,(ST + U) \\ &= 4BDT(ST + U)/W \\ &= 4(2.5\ R_y)\ DT(ST + U)/W \end{aligned}$$

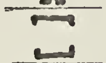
Finally substituting

$$D = BS = 2.5\ R_y\ S\ ,$$

the expression for A becomes

$$\begin{aligned} A &= 4(2.5\ R_y)^2\ ST(ST + U)/W \\ &= \left[\ 25\ ST\ (ST + U)/W \right]\ R^2 \end{aligned}$$

Table B-1. Section Formulas Used by Program

Section No.	Profile	$A = f(R, W, S, T, U)$	Remarks
1		$A = 20.5\ R^2/W$	$D = B$
2		$A = 10.9\ (S^2 + S)\ R^2/W$	$D > B$
3		$A = 41.7\ (0.6\ S^2 + S)\ R^2/W$	$t_w = 0.6\ t_f$
4		$A = 12.57\ R^2$	
5		$A = 41.0\ R^2/W$	$D = 2B$
6		$A = 12.5\ (2S + 1)\ R^2/W$	
7		$A = 5.8\ (S^2 + 2S)\ R^2/W$	
8		$A = 25\ (S + 2)\ R^2/W$	$t_w = t_f$
9		$A = 8(1.2S + 2)\ R^2/W$	$t_w = 1.2\ t_f$
10		$A = 8(0.7S + 2)\ R^2/W$	$t_w = 0.7\ t_f$
11		$A = 8(0.6S + 2)\ R^2/W$	(WF); $t_w = 0.6t_f$
12		$A = 25(S + 1)\ R^2/W$	$t_w = t_f$
13		$A = 15.7\ ST\ (2ST + 1)\ R^2/W$	"
14		"	"
15		$A = 25\ ST(ST + U)\ R^2/W$	

Note: Weak axis is considered vertical in all profile sketches

Section formulas for the other 14 shapes programmed are derived in a similar manner, and are tabulated in Table B-1.

The use of these equations in the computer program is of two types. The function sub-program OPT2 computes A, given the arguments R, W, S, T, and U. Function RGF is the inverse of OPT2, solving the equations for R, given A, W, S, T, and U. The appropriate one of these two functions is called to compute the second coordinate of the intersection point once the other has been found in some different manner.

Procedures for location of intersection of curves

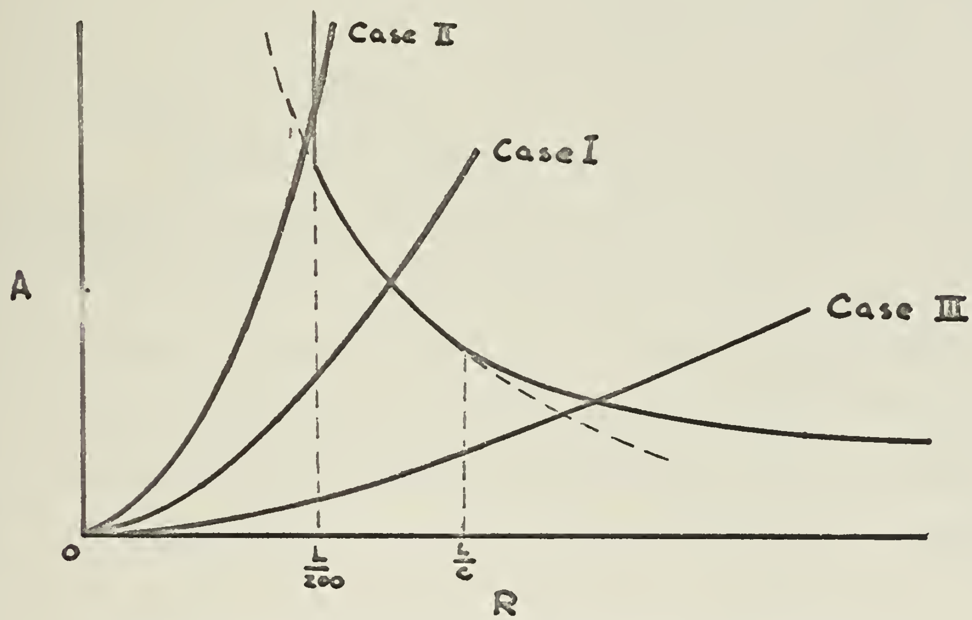


Fig. B-2 $A_{reqd.}$ curve, compression members

a. Equation of vertical segment of $A_{reqd.}$ curve:

$$R = \frac{L}{200}$$

b. Equation of hyperbolic segment:

$$A = \frac{P}{F_a} = \frac{PL^2}{149,000\, R^2}$$

c. Equation of cubic segment:

$$A = \frac{P}{F_a} = \frac{\frac{5P}{3} + \left(\frac{3PL}{8C}\right)\frac{1}{R} - \left(\frac{PL^3}{8C^3}\right)\frac{1}{R^3}}{F_y - \left(\frac{F_y L^2}{2C^2}\right)\frac{1}{R^2}} \times \frac{24C^3R^3}{24C^3R^3}$$

$$= \frac{(40PC^3)\, R^3 + (9PLC^2)\, R^2 - 3PL^3}{(24\, F_y\, C^3)\, R^3 - (12\, F_y\, L^2\, C)\, R}$$

where F_y is in ksi,

and $C = \sqrt{\frac{2\pi^2 E}{F_y}} = \sqrt{\frac{573,000}{F_y}}$

d. Method of solution:

Reference to Figure B-2 above will show that the optimum design point may fall on any one of the three segments described. The exact location is dependent upon P and L, which position the entire $A_{reqd.}$ curve vertically and horizontally, respectively, in the A-R region, and upon the "spread" of the parabola determined by the section formula. Since imaginary extensions of the hyperbolic segment (shown dotted in Fig. B-2) theoretically cover a range of R-values from 0 to ∞, it is logical to use the hyperbola as a starting point.

A function sub-program (OPT1) is provided, which solves simultaneously the hyperbolic equation (b, above) and the appropriate section formula from Table B-1, producing the A-coordinate of the parabola-hyperbola intersection.

R is then back-figured from the hyperbolic formula. A series of tests then determines the horizontal location of R, with respect to the end points of the hyperbolic segment ($\frac{L}{200}$ and $\frac{L}{C}$). If R is found to lie on or between these points (Case I), the point desired is given by the (A, R) coordinates just determined. If $R < \frac{L}{200}$ (Case II), the value of $R_{opt.}$ is taken as $\frac{L}{200}$, and $A_{opt.}$ is computed by function OPT2, which is merely the section formula expression of A as a function of R.

Case III on the graph of Figure B-2 presents a more difficult problem. There, the R-coordinate of the parabola-hyperbola intersection is found to be greater than L/C, indicating that the actual optimum lies at the parabola-cubic intersection. The fact that the hyperbola extension drops below the cubic curve to the right of their point of tangency (R = L/C) is of great value in determining the location of the desired root of the fifth-degree equation, which results from simultaneous solution of the parabolic and cubic curves. This is to say, the desired root is known to be greater than the false root lying on the hyperbola extension.

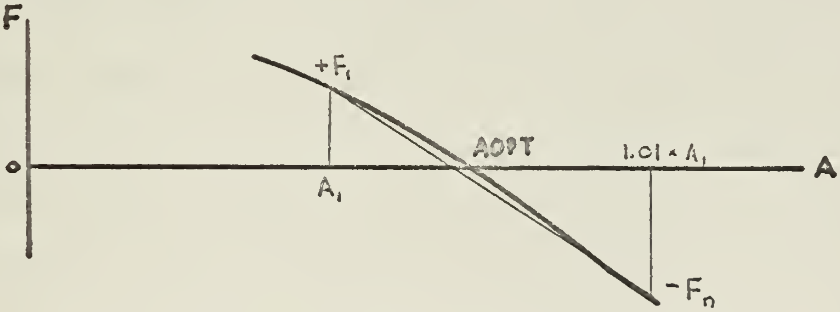


Fig. B-3 Generalized polynomial function.

Figure B-3 represents a portion of the graph of a general polynomial function with ordinates given by F and arguments by A. The homogeneous fifth-degree equation in question can be similarly represented. Knowing

that we have a reasonable approximation to the true root, and that it lies to the left of this intercept (AOPT), a simple iterative process can be employed to locate the true intercept. Starting with the false root A_1, the value of the function, F_1, is computed. The argument is increased by 0.01 of its present value and the function re-computed. (This is performed by the sub-program function OPT3.) This cycle is repeated until the algebraic sign of the ordinate changes. The true intercept is then computed by linear interpolation between the two points for which the ordinate changed sign. It is believed that this simple iterative approach is valid for most cases which might arise. The comparatively small increment used tends to preclude the skipping of one or more intercepts, including the desired root. (If this should happen, it would be readily recognizable by the extremely high values of AOPT and ROPT produced.)

A case arose during the testing of the program in which the false root determined by the parabola-hyperbola intersection lay so close to the actual root that round-off error within the machine caused the iteration to begin at a point greater than the true root. To prevent recurrence of this, a test was inserted, by which the argument is incremented in steps as before. At the end of ten cycles if the sign has not changed, a test is carried out to determine the tendency of the curve toward or away from the axis. In the later case, the iteration reverses direction and proceeds until a root is encountered. Once the A-coordinate of the intersection has been determined, R is back-figured by RGF.

C. Documentation of Computer Program

General description

The program is written in the FORTRAN language and was developed for the IBM 7094 Data Processing System. Any modifications necessary for adaptation to other systems are not herein discussed. The only restriction on the size of truss which can be handled is the size of the connectivity and member-width matrices, as specified in the first two statements of the source program. In any new compilation of the program these statements should be revised to read:

```
DIMENSION   MCONN (J, M)
DIMENSION WJTMIN (J), WJTMAX (J)
```

where J = no. of joints in truss

M = no. of members in truss.

Input data

The input data consists of three parts:

1) Truss size and method of connection (one card)

2) Connectivity data (one card per member)

3) Section properties (one card per member, plus one card for a dummy member, to terminate the processing of members).

The information required for each card is as follows:

1) Truss card (1 only)

NO = identification number for design being evaluated

JTS = no. of joints in structure

MBRS = no. of members in structure

CONN = the word WELDED or the word RIVETS

2) Connectivity card (1 per member)

N = identification number for member (These must be in correct order with no omissions).

JNEG = number of joint at "negative end"

JPOS = number of joint at "positive end"

JSHAPE = identification number for section type (see Table B-1 for list)

PSI = 0. or 90., the number of degrees in the angle between the weak axis of the section and the plane of the truss.

3) Section property card (1 per member)

N = identification number for member

D = overall depth, in inches

B = overall width, in inches

D1 = depth of sub-element for a built-up section

B1 = width of sub-element for a built-up section

JSHAPE = shape number, as defined above

FY = yield stress, in ksi

A1 = cross sectional area in $in.^2$

SI2 = sectional moment of inertia about the I_2 or major axis of inertia, in $in.^4$

SI3 = sectional moment of inertia about the I_3 or

minor axis of inertia, in. $in.^4$

WTMAX = the maximum width-thickness ratio existing for plate projections of the section, as defined below:

WF - and I-sections: $\frac{1}{2}\ \frac{w_f}{t_f}$

T-sections: $\frac{depth}{t_w}$

Channels: $\frac{w_f}{t_f}$

Round sections: 1.0

XL = length of member, in inches

P = axial load, in kips

PSI = orientati on angle of major axis, as defined above.

Careful attention must be given when punching Data cards, to following exactly the field specifications for input given in FORMAT statements numbers 1, 7 and 23.

Sample blocks of data are given in Appendix D, on illustrative problems.

Output data

Output data format is controlled by the program. Sample output may be found in Appendix D.

Program listing

(See next page)

```
C     TRUSS DESIGN EVALUATION PROGRAM
C     THOMAS H. OSWALD, LT, USN       MIT       OCTOBER, 1963

C     MAIN PROGRAM
      DIMENSION MCONN(25, 25)
      DIMENSION WJTMIN(25), WJTMAX(25)
C     READ DATA FOR SIZE OF TRUSS AND METHOD OF CONNECTION
1     FORMAT (I3, 10X, I3, 10X, I3, 10X, A6)
2     READ 1, NO, JTS, MBRS, CONN

C     PRINT OUTPUT HEADINGS
3     FORMAT (11H1DESIGN NO., I3)
4     FORMAT (1H0)
5     FORMAT (1H0,23X,23HTRUSS MEMBER EVALUATION)
6     FORMAT (1H0, 41X, 2HVR, 3X, 5HAREQD, 2X, 5HRREQD, 3X, 4HAOPT,
     1  3X, 4HROPT, 3X, 3HWMF)
7     FORMAT (I3, 10X, I3, 10X, I3, 10X, I3, 10X, F5.1)
      PRINT 3, NO
      PRINT 4
      PRINT 5
      PRINT 6

C     ASSEMBLE CONNECTIVITY MATRIX
C        MATRIX ELEMENT IS A 1 FOR SINGLE-PLANE MEMBER
C        MATRIX ELEMENT IS A 2 FOR DOUBLE-PLANE MEMBER

      DO 15          M = 1, MBRS
      READ 7, N, JNEG, JPOS, JSHAPE, PSI
      IF (JSHAPE - 7)            8, 8, 9
8     L = 1
      GO TO 14
9     IF (JSHAPE - 13)           11, 10, 10
10    L = 2
      GO TO 14
11    IF (PSI)                   13, 12, 13
12    L = 1
      GO TO 14
13    L = 2
14    MCONN(JNEG, M) = L
15    MCONN (JPOS, M) = L

C     COUNT NUMBERS OF JOINTS HAVING ALL SINGLE-PLANE AND ALL
C        DOUBLE-PLANE MEMBERS
      MCNT11 = 0
      MCNT22 = 0
      DO 22          J = 1, JTS
      MCNT1 = 0
      MCNT2 = 0
      DO 18          M = 1, MBRS
      IF (MCONN(J,M) - 1)        18, 16, 17
16    MCNT1 = MCNT1 + 1
      GO TO 18
17    MCNT2 = MCNT2 + 1
18    CONTINUE
```

```
      IF (MCNT2)                     20, 19, 20
19    MCNT11 = MCNT11 + 1
      GO TO 22
20    IF (MCNT1)                     22, 21, 22
21    MCNT22 = MCNT22 + 1
22    CONTINUE

23    FORMAT (I2, 4F6.2, I3, F4.0, 3F6.2, F5.1, F5.0, F7.2, F4.0)
24    FORMAT (7HOMEMBER,I4,12H ZERO STRESS)
25    FORMAT (7HOMEMBER, I4, 17H EXCEEDS (B/T)MAX, 12X, F5.2)
26    FORMAT (7HOMEMBER, I4, 25H COMPR., EXCEEDS (L/R)MAX,
     1  4X, F5.2, 9X, F5.2)
27    FORMAT (7HOMEMBER, I4, 21H COMPR., OVERSTRESSED,
     1  8X, F5.2, 2X, F5.2)
28    FORMAT (7HOMEMBER, I4, 26H TENSION, EXCEEDS (L/R)MAX,
     1  3X, F5.2, 9X, F5.2)
29    FORMAT (7HOMEMBER, I4, 22H TENSION, OVERSTRESSED,
     1  7X, F5.2, 2X, F5.2)
30    FORMAT (7HOMEMBER, I4, 15H OPTIMUM DESIGN,
     1  35X, F5.2, 2X, F5.2, 2X, F5.2)
C     INITIALIZE COUNTING VARIABLES
      JCNT1  = 0
      JCNT2  = 0
      JCNT3  = 0
      JCNT4  = 0
      JCNT5  = 0
      JCNT6  = 0
      JCNT7  = 0
      JCNT8  = 0
      JCNT9  = 0
      JCNT10 = 0
      JCNT11 = 0
C     INITIALIZE CUMULATIVE VOLUMES OF ACTUAL AND IDEALIZED TRUSSES
      TVOL = 0.
      VOPT = 0.

C     READ SECTION PROPERTIES FOR ONE MEMBER
31    READ 23, N, D, B, D1, B1, JSHAPE, FY, A1, SI2, SI3, WTMAX, XL, P,
     1 PSI
C     IF ALL MEMBERS HAVE BEEN PROCESSED, PROCEED TO TRUSS SUMMARY
C     IF NOT, BEGIN SPECIFICATION CHECK FOR MEMBER
      IF (N - MBRS)                  32, 32, 114
32    JCNT1 = JCNT1 + 1
C     ADD VOLUME OF MEMBER TO CUMULATIVE TRUSS VOLUME
      TVOL = TVOL + A1*XL
C     COMPUTE PROFILE PROPORTIONS FOR MEMBER
      S = D / B
      T = D1 / D
      U = B1 / B
C     DETERMINE WIDTH OF MEMBER NORMAL TO PLANE OF TRUSS
      IF (PSI)                       34, 33, 34
33    THICK = B
      GO TO 35
34    THICK = D
```

```
C     STORE GREATEST AND LEAST MEMBER WIDTHS FOR EACH JOINT
35    DO 44          J = 1, JTS
      IF (MCONN(J,N) - 2)            44, 36, 44
36    IF (WJTMIN(J))                 38, 37, 38
37    WJTMIN(J) = THICK
      GO TO 44
38    IF (THICK - WJTMIN(J))         39, 39, 41
39    IF (WJTMIN(J) - WJTMAX(J))     43, 40, 40
40    WJTMAX(J) = WJTMIN(J)
      WJTMIN(J) = THICK
      GO TO 44
41    IF (THICK - WJTMAX(J))         44, 44, 42
42    WJTMAX(J) = THICK
      GO TO 44
43    WJTMIN(J) = THICK
44    CONTINUE

C     COMPUTE (L/R)MAX FOR MEMBER
      IF (SI2-SI3)                   45, 45, 46
45    RMIN = SQRTF(SI2/A1)
      GO TO 47
46    RMIN = SQRTF(SI3/A1)
47    SRMAX = XL/RMIN

C     SET SHAPE FACTOR FOR CROSS SECTION
      GO TO (48,48,49,52,48,48,48,48,50,50,50,48,48,48,48), JSHAPE
48    SF = 0.43
      GO TO 51
49    SF = 1.28
      GO TO 51
50    SF = 0.70
C     COMPUTE ALLOWABLE (B/T)
51    WTRAL = WTF(SF,FY)
      GO TO 53
52    WTRAL = WTMAX

C     COMPUTE AVERAGE AXIAL STRESS
53    STRESS = P / A1
      IF (STRESS)                    55, 54, 65

C     ZERO STRESS
54    PRINT 24, N
      PRINT 4
      JCNT3 = JCNT3 + 1
      GO TO 31

C     COMPRESSION MEMBER
C     CHECK (B/T)
55    IF (WTMAX - WTRAL)             57, 57, 56
56    VR = WTMAX / WTRAL
      PRINT 25, N, VR
      JCNT2 = JCNT2 + 1
```

```
C       SET ALLOWABLE (L/R)
57      SRALL = 200.
C       MINIMUM RADIUS OF GYRATION FOR THIS LENGTH
        RREQD = XL/SRALL
C       CHECK (L/R)
        IF (SRMAX-SRALL)              60, 60, 58
58      VR = SRMAX/SRALL
59      PRINT 26, N, VR, RREQD
        JCNT4 = JCNT4 + 1
        GO TO 75

C       COMPUTE ALLOWABLE COMPRESSIVE STRESS
60      CSTRAL = CSF(SRMAX,FY)
C       MINIMUM AREA FOR THIS LOAD AND L/R
        AREQD = -P/CSTRAL
C       CHECK (P/A)
        IF ((-STRESS)/CSTRAL - 1.0)  63, 64, 61
61      VR = (-STRESS)/CSTRAL
62      PRINT 27, N, VR, AREQD
        JCNT5 = JCNT5 + 1
        GO TO 75
63      JCNT8 = JCNT8 + 1
        GO TO 75

64      JCNT9 = JCNT9 + 1
        GO TO 75
C       TENSION MEMBER
C       IF TRUSS IS RIVETED, INCREASE P BY 100/85 TO SIMULATE
C          15-PERCENT REDUCTION OF AREA
B65     IF (CONN * 264612521415)   67, 66, 67
66      P = 100. * P / 85.
        STRESS = P / A1

C       SET ALLOWABLE (L/R)
67      SRALL = 240.
C       MINIMUM RADIUS OF GYRATION FOR THIS LENGTH
        RREQD = XL/SRALL
C       CHECK (L/R)
        IF (SRMAX-SRALL)              70, 70, 68
68      VR = SRMAX/SRALL
69      PRINT 28, N, VR, RREQD
        JCNT6 = JCNT6 + 1

C       COMPUTE ALLOWABLE TENSILE STRESS
70      TSTRAL = TSF(FY)
C       MINIMUM AREA FOR THIS LOAD
        AREQD = P/TSTRAL
C       CHECK (P/A)
        IF (STRESS/TSTRAL - 1.0)   73, 74, 71
71      VR = STRESS/TSTRAL
72      PRINT 29, N, VR, AREQD
        JCNT7 = JCNT7 + 1
        GO TO 109
73      JCNT10 = JCNT10 + 1
        GO TO 109
74      JCNT11 = JCNT11 + 1
        GO TO 111
```

```
C     BEGIN OPTIMIZATION ROUTINE

C     COMPRESSION MEMBERS
C     LOCATE INTERSECTION OF PARABOLA AND HYPERBOLA
75    C = SQRTF(573000. / FY)
      AOPT1 = OPT1(JSHAPE, XL, P, WTRAL, S, T, U)
C     LOCATE R OF INTERSECTION WITH RESPECT TO L/C AND L/200
      ROPT1 = SQRTF(-P*XL**2 /(149000.*AOPT1))
      IF (ROPT1 - XL/C)            76, 76, 79
76    IF (ROPT1 - XL/200.)         78, 77, 77
C     PARABOLA INTERSECTS HYPERBOLIC SEGMENT OF AR CURVE
77    AOPT = AOPT1
      ROPT = ROPT1
      GO TO 108
C     PARABOLA INTERSECTS (L/R = 200) VERTICAL
78    ROPT = XL / 200.
      AOPT = OPT2(JSHAPE, ROPT, WTRAL, S, T, U)
      GO TO 108
C     PARABOLA INTERSECTS CUBIC SEGMENT OF AR CURVE
C     COMPUTE 5TH-DEGREE EQUATION CONSTANTS COMMON TO ALL SECTIONS
79    ALPHA = SQRTF(WTRAL)
      C1 = 24. * FY * C**3 * WTRAL * ALPHA
      C2 = FY * XL**2 * C * ALPHA
      C3 = 40. * (-P) * C**3 * WTRAL * ALPHA
      C4 = (-P) * XL * C**2 * WTRAL
      C5 = (-P) * XL**3
C     COMPUTE CONSTANTS DEPENDING UPON THE SECTION SHAPE
80    GO TO (81,82,83,84,85,86,87,88,89,90,91,92,93,93,95), JSHAPE
81    BETA = SQRTF(20.5)
      GO TO 96
82    BETA = SQRTF(10.9* (S**2 + S) )
      GO TO 96
83    BETA = SQRTF(41.7* (.6*S**2 + S) )
      GO TO 96
84    BETA = SQRTF(12.6)
      GO TO 96
85    BETA = SQRTF(41.0)
      GO TO 96
86    BETA = SQRTF(25.*S + 12.5)
      GO TO 96
87    BETA = SQRTF(5.8 * (S**2 + 2. * S))
      GO TO 96
88    BETA = SQRTF(25.*S + 50.)
      GO TO 96
89    BETA = SQRTF(9.6*S + 16.)
      GO TO 96
90    BETA = SQRTF(5.6*S + 16.)
      GO TO 96
91    BETA = SQRTF(4.8*S + 16.)
      GO TO 96
92    BETA = SQRTF(25.*S + 25.)
      GO TO 96
93    BETA = SQRTF(15.7 * S * T *(2.*S*T + 1.))
      GO TO 96
95    BETA = SQRTF(25. * S * T * (S * T + U) )
96    C8 = 12. * BETA**2
      C9  =  9. * BETA
      C10 =  3. * BETA**3
```

```
C     ITERATION TO LOCATE INTERSECTION OF PARABOLA AND CUBIC CURVE
97    AOLD = AOPT1
      FOLD = OPT3(AOLD, C1, C2, C3, C4, C5, C8, C9, C10)
      Z = 1.01
      IF (FOLD)               98, 107, 98
98    FOOLD = FOLD
      DO 102   I = 1, 10
      ANEW = Z * AOLD
      FNEW = OPT3(ANEW, C1, C2, C3, C4, C5, C8, C9, C10)
      IF (FNEW)               99, 106, 100
99    IF (FOLD)              101, 107, 105
100   IF (FOLD)              105, 107, 101
101   AOLD = ANEW
102   FOLD = FNEW
      IF (FNEW/FOOLD - 1.)    98, 98, 103
103   Z = 0.99
      GO TO 98
105   AOPT = AOLD + ABSF(FOLD/(FNEW-FOLD)) * (ANEW-AOLD)
      ROPT = RGF(JSHAPE, AOPT, WTRAL, S, T, U)
      GO TO 108
106   AOPT = ANEW
      ROPT = RGF(JSHAPE, AOPT, WTRAL, S, T, U)
      GO TO 108
107   AOPT = AOLD
      ROPT = RGF(JSHAPE, AOPT, WTRAL, S, T, U)

C     ADD VOLUME OF OPTIMUM MEMBER TO VOLUME OF OPTIMUM TRUSS
108   VOPT = VOPT + AOPT*XL
C     COMPUTE WEIGHT MERIT FACTOR FOR MEMBER
      WMF = A1 / AOPT
      GO TO 113

C     OPTIMIZATION ROUTINE, TENSION MEMBERS
109   AOPT = P / TSTRAL
110   ROPT = XL/240.
      GO TO 112
111   AOPT = A1
      ROPT = XL / 240.

C     INCREMENT VOLUME OF OPTIMUM TRUSS
112   VOPT = VOPT + AOPT*XL
      WMF = A1 / AOPT
113   PRINT 30, N, AOPT, ROPT, WMF
      PRINT 4

C     NEXT MEMBER
      GO TO 31

C     ALL MEMBERS PROCESSED
C     COMPUTE WEIGHT MERIT FACTOR FOR TRUSS
114   WMFT = TVOL / VOPT

C     PRINT MEMBER SUMMARY
115   FORMAT (1H1, 23X, 24HSUMMARY FOR ENTIRE TRUSS)
116   FORMAT (27H0EVALUATION OF TRUSS WEIGHT)
117   FORMAT (1H0, 33X, 7HTENSION, 6X, 11HCOMPRESSION)
```

```
118   FORMAT (31HOTOTAL NO. OF MEMBERS EVALUATED, 33X, I3)
119   FORMAT (27HOMEMBERS EXCEEDING (B/T)MAX, 24X, I3)
120   FORMAT (27HOMEMBERS EXCEEDING (L/R)MAX, 9X, I3, 12X, I3)
121   FORMAT (27HOMEMBERS HAVING ZERO STRESS, 37X, I3)
122   FORMAT (21HOOVERSTRESSED MEMBERS, 15X, I3, 12X, I3)
123   FORMAT (22HOUNDERSTRESSED MEMBERS, 14X, I3, 12X, I3)
124   FORMAT (23HOFULLY STRESSED MEMBERS, 13X, I3, 12X, I3)
125   FORMAT (28HOTRUSS WEIGHT MERIT FACTOR =, F7.3)
      PRINT 115
      PRINT 4
      PRINT 116
      PRINT 117
      PRINT 118, JCNT1
      PRINT 119, JCNT2
      PRINT 120, JCNT6, JCNT4
      PRINT 121, JCNT3
      PRINT 122, JCNT7, JCNT5
      PRINT 123, JCNT10, JCNT8
      PRINT 124, JCNT11, JCNT9
      PRINT 4
      PRINT 125, WMFT

C     PRINT JOINT SUMMARY
126   FORMAT (31H1EVALUATION OF JOINT COMPLEXITY)
1261  FORMAT (30HOTOTAL NO. OF JOINTS EVALUATED, 23X, I3)
127   FORMAT (50HONO. OF JOINTS REQUIRING SINGLE GUSSET PLATES ONLY,
     1 3X, I3)
128   FORMAT (50HONO. OF JOINTS REQUIRING DOUBLE GUSSET PLATES ONLY,
     1 3X, I3)
129   FORMAT (41HONO. OF JOINTS REQUIRING MIXED CONNECTION, 12X, I3)
      PRINT 126
      PRINT 1261, JTS
      PRINT 127, MCNT11
      PRINT 128, MCNT22
      JPCT = JTS - MCNT11 - MCNT22
      PRINT 129, JPCT
130   FORMAT (57H3MEMBER WIDTH RATIOS FOR JOINTS WITH DOUBLE GUSSET PLAT
     1ES)
131   FORMAT (6HOJOINT, I3, 10X, F4.2)
      PRINT 130

C     COMPUTE AND PRINT RATIO OF WIDEST TO NARROWEST MEMBER AT EACH
C        DOUBLE-GUSSET JOINT
      DO 136       J = 1, JTS
      IF (WJTMIN(J))              132, 136, 132
132   IF (WJTMAX(J))              134, 133, 134
133   WRATIO = 1.0
      GO TO 135
134   WRATIO = WJTMAX(J) / WJTMIN(J)
135   PRINT 131, J, WRATIO
136   CONTINUE

C     NEXT TRUSS
      GO TO 2
      END
```

```
C     FUNCTION SUBPROGRAMS
C     ALLOWABLE WIDTH-THICKNESS RATIO
      FUNCTION WTF(S,F)
      WTF = 113.8*SQRTF(S/F)
      RETURN
      END

C     ALLOWABLE COMPRESSIVE STRESS
      FUNCTION CSF(X,Y)
      C = SQRTF(573000./Y)
      IF (X-C)     1,1,2
1     FS = 5./3. + (3.*X) / (8.*C) - (X**3) / (8.*C**3)
      CSF = (Y*(1.-(X**2 / (2.*C**2) ) ) ) / FS
      RETURN
2     CSF = 149000. / X**2
      RETURN
      END

C     ALLOWABLE TENSILE STRESS
      FUNCTION TSF(X)
      TSF = 0.6*X
      RETURN
      END

C     INTERSECTION OF PARABOLA AND HYPERBOLA
      FUNCTION OPT1(JSHAPE, XL, P, W, S, T, U)
      GO TO (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 13, 15), JSHAPE
1     C6 = .0117
      FSTU = 1.
      GO TO 100
2     C6 = .0086
      FSTU = S + S**2
      GO TO 100
3     C6 = .0167
      FSTU = S + 0.6*S**2
      GO TO 100
4     C6 = .0092
      FSTU = 1.
      GO TO 100
5     C6 = .0166
      FSTU = 1.
      GO TO 100
6     C6 = .0092
      FSTU = 1. + 2.*S
      GO TO 100
7     C6 = .0062
      FSTU = S**2 + 2.*S
      GO TO 100
8     C6 = .0130
      FSTU = S + 2.
      GO TO 100
```

```
9       FSTU = 1.2*S + 2.
        GO TO 99
10      FSTU = 0.7*S + 2.
        GO TO 99
11      FSTU = 0.6*S + 2.
99      C6 = .0073
        GO TO 100
12      C6 = .0130
        FSTU = 1. + S
        GO TO 100
13      C6 = .0103
        FSTU = S * T * (2.*S*T + 1.)
        GO TO 100
15      C6 = .0130
        FSTU = S*T*(S*T + U)
100     OPT1 = C6*XL*SQRTF((-P)*FSTU / W)
        RETURN
        END


C       AREA AS A FUNCTION OF R
        FUNCTION OPT2(JSHAPE, R, W, S, T, U)
        GO TO (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 13, 15), JSHAPE
1       OPT2 = 20.5 * R**2 / W
        RETURN
2       OPT2 = 10.9 * S * (1.+S) * R**2 / W
        RETURN
3       OPT2 = 41.7 * S * (1.+.6*S) * R**2 / W
        RETURN
4       OPT2 = 12.6 * R**2
        RETURN
5       OPT2 = 41.0 * R**2 / W
        RETURN
6       OPT2 = (12.5 + 25.*S) * R**2 / W
        RETURN
7       OPT2 = 5.8 * (S**2 + 2.*S) * R**2 / W
        RETURN
8       OPT2 = 25. * (S+2.) * R**2 / W
        RETURN
9       OPT2 = (9.6*S + 16.) * R**2 / W
        RETURN
10      OPT2 = (5.6*S + 16.) * R**2 / W
        RETURN
11      OPT2 = (4.8*S + 16.) * R**2 / W
        RETURN
12      OPT2 = (25. + 25.*S) * R**2 / W
        RETURN
13      OPT2 = 15.7 * S * T * (2.*S*T + 1.) * R**2 / W
        RETURN
15      OPT2 = 25. * S * T * (S*T + U) * R**2 / W
        RETURN
        END
```

```
C       HOMOGENEOUS 5TH-DEGREE EQUATION FOR INTERSECTION OF
C          PARABOLA AND CUBIC
        FUNCTION OPT3 (A, C1, C2, C3, C4, C5, C8, C9, C10)
        ALPHA = SQRTF(A)
        OPT3 = C1 * ALPHA**5 - (C2*C8+C3) * ALPHA**3 - C4*C9*A + C5*C10
        RETURN
        END


C       RADIUS OF GYRATION AS A FUNCTION OF A
        FUNCTION RGF(JSHAPE, A, W, S, T, U)
        GO TO (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 13, 15), JSHAPE
1       RGF = SQRTF(A * W / 20.5)
        RETURN
2       RGF = SQRTF(A * W / (10.9 * S * (1. + S) ) )
        RETURN
3       RGF = SQRTF(A * W / (41.7 * S * (1. + .6*S) ) )
        RETURN
4       RGF = SQRTF(A / 12.6)
        RETURN
5       RGF = SQRTF(A * W / 41.0)
        RETURN
6       RGF = SQRTF(A * W / (12.5 * (1. + 2.*S) ) )
        RETURN
7       RGF = SQRTF(A * W / (5.8 *(S**2 + 2.*S) ) )
        RETURN
8       RGF = SQRTF(A * W / (25. * (S+2.) ) )
        RETURN
9       RGF = SQRTF(A * W / (8. * (1.2*S + 2.) ) )
        RETURN
10      RGF = SQRTF(A * W / (8. * (0.7*S + 2.) ) )
        RETURN
11      RGF = SQRTF(A * W / (8. * (0.6*S + 2.) ) )
        RETURN
12      RGF = SQRTF(A * W / (25. * (1. + S) ) )
        RETURN
13      RGF = SQRTF(A * W / (15.7 * S * T *(2.*S*T + 1.) ) )
        RETURN
15      RGF = SQRTF(A * W / (25.*T*S*(U + T*S) ) )
        RETURN
        END




C       END OF COMPLETE PROGRAM
```

D Illustrative Problems

In the following pages are presented sketches and input-output data for two test trusses. The first, Truss No. 1, is statically determinate, therefore only one analysis cycle was necessary. Data is presented for an initial set of member designs and for one cycle of redesign.

Truss No. 2 is statically indeterminate and hence requires re-analysis after each cycle of design. Data format for the analysis phases is that of the STRESS system. Input data for the evaluation portions conforms exactly to the FORMAT statements found in the program. Output data is compressed from the 120-characters-per-line width of the on-line printer to the 80-characters-per-line width of punched cards, for convenience in printing this section. Therefore spacing between columns of evaluation output will not be found to agree with output FORMAT statements.

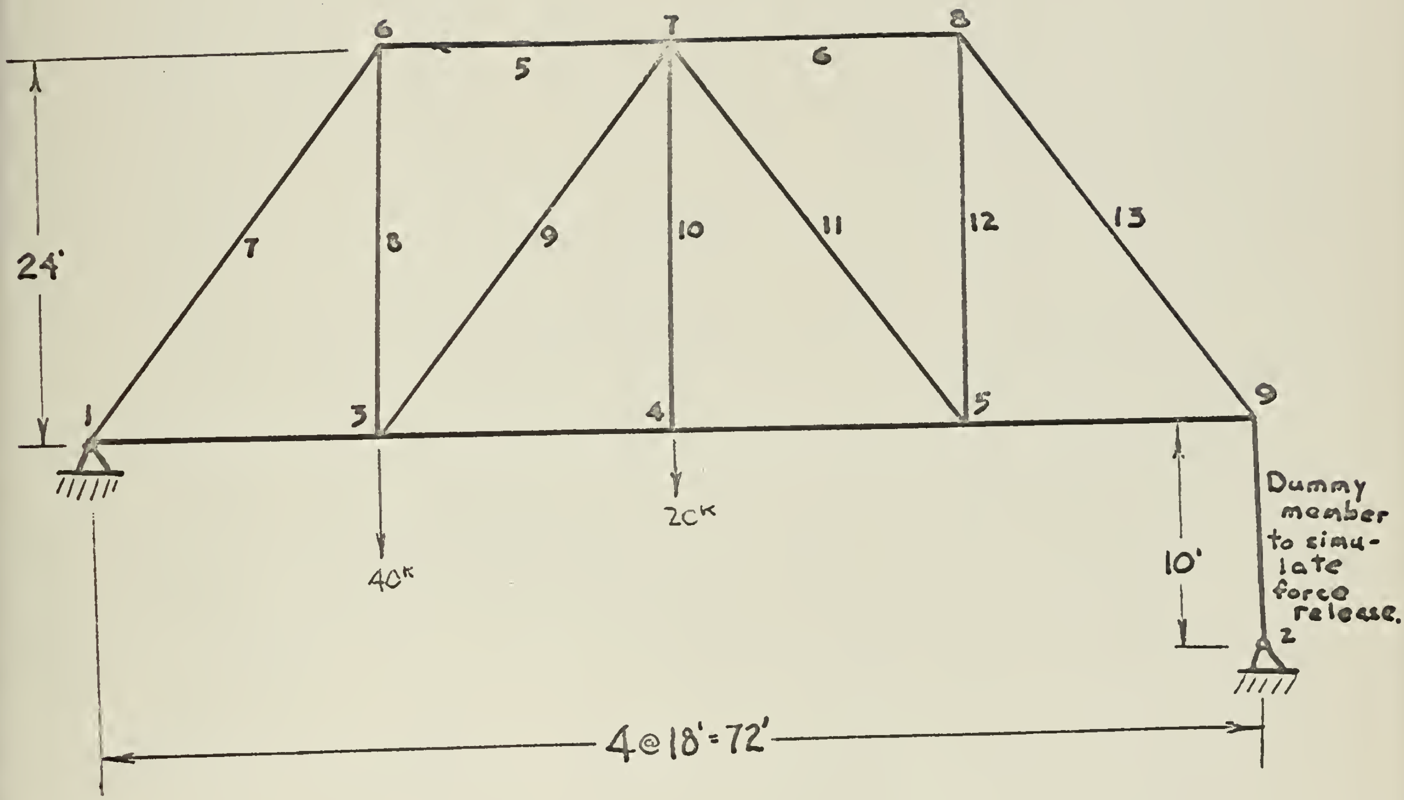


Fig. D-1 Statically determinate test truss.

STRESS INPUT FOR DETERMINATE TRUSS OF FIG. D-1

```
STRUCTURE   STATICALLY DETERMINATE TRUSS
NUMBER OF JOINTS   9
NUMBER OF SUPPORTS 2
NUMBER OF MEMBERS 14
TYPE   PLANE TRUSS
METHOD   STIFFNESS
JOINT 1 COORDINATES      0.      0.
JOINT 2 COORDINATES    864.   -120.
JOINT 3 COORDINATES    216.      0.
JOINT 4 COORDINATES    432.      0.
JOINT 5 COORDINATES    648.      0.
JOINT 6 COORDINATES    216.    288.
JOINT 7 COORDINATES    432.    288.
JOINT 8 COORDINATES    648.    288.
JOINT 9 COORDINATES    864.      0.
MEMBER  1 FROM 1 TO 3, PRISMATIC, 1.00
MEMBER  2 FROM 3 TO 4, PRISMATIC, 1.00
MEMBER  3 FROM 4 TO 5, PRISMATIC, 1.00
MEMBER  4 FROM 5 TO 9, PRISMATIC, 1.00
MEMBER  5 FROM 6 TO 7, PRISMATIC, 1.00
MEMBER  6 FROM 7 TO 8, PRISMATIC, 1.00
MEMBER  7 FROM 1 TO 6, PRISMATIC, 1.00
MEMBER  8 FROM 3 TO 6, PRISMATIC, 2.38
MEMBER  8 FROM 3 TO 6, PRISMATIC, 1.00
MEMBER  9 FROM 3 TO 7, PRISMATIC, 3.14
MEMBER  9 FROM 3 TO 7, PRISMATIC, 1.00
MEMBER 10 FROM 4 TO 7, PRISMATIC, 1.00
MEMBER 11 FROM 7 TO 5, PRISMATIC, 1.00
MEMBER 12 FROM 5 TO 8, PRISMATIC, 1.00
MEMBER 13 FROM 8 TO 9, PRISMATIC, 1.00
MEMBER 14 FROM 9 TO 2, PRISMATIC, 1.00
NUMBER OF LOADINGS   1
LOADING   ARBITRARY
JOINT 3 LOADS   0.    -40.
JOINT 4 LOADS   0.    -20.
PRINT MEMBER FORCES
SOLVE
```

STRESS OUTPUT FOR DETERMINATE TRUSS

STRUCTURE STATICALLY DETERMINATE TRUSS

LOADING ARBITRARY

MBR	JNT	AXIAL FORCE
1	1	-30.0000017
1	3	30.0000017
2	3	-30.0000012
2	4	30.0000012
3	4	-29.9999990
3	5	29.9999990
4	5	-15.0000038
4	9	15.0000038
5	6	30.0000000
5	7	-30.0000000
6	7	14.9999924
6	8	-14.9999924
7	1	50.0000010
7	6	-50.0000010
8	3	-40.0000038
8	6	40.0000038
9	3	0.0000025
9	7	-0.0000025
10	4	-20.0000010
10	7	20.0000010
11	7	24.9999940
11	5	-24.9999940
12	5	-20.0000014
12	8	20.0000014
13	8	24.9999986
13	9	-24.9999986
14	9	19.9999983
14	2	-19.9999983

NOTE - SECOND FIGURE GIVES SIGN OF FORCE.

EVALUATION INPUT - DETERMINATE TRUSS - FIRST DESIGN

15	8	13	WELDED	
1	1	3	9	0.0
2	3	4	9	0.0
3	4	5	9	0.0
4	5	2	9	0.0
5	6	7	11	90.0
6	7	8	11	90.0
7	1	6	13	0.0
8	3	6	1	0.0
9	3	7	4	0.0
10	4	7	2	0.0
11	7	5	13	0.0
12	5	8	2	0.0
13	8	2	13	0.0

1	3.00	2.33	3.00	2.33	9	36.	1.64	.46	2.5	4.5	216.	30.00	0.
2	3.00	2.33	3.00	2.33	9	36.	1.64	0.46	2.5	4.5	216.	30.00	0.
3	3.00	2.33	3.00	2.33	9	36.	1.64	0.46	2.5	4.5	216.	30.00	0.
4	3.00	2.33	3.00	2.33	9	36.	1.64	0.46	2.5	4.5	216.	15.00	0.
5	5.00	5.00	5.00	5.00	11	36.	4.70	7.51	21.30	7.0	216.	-30.00	90.
6	5.00	5.00	5.00	5.00	11	36.	4.70	7.51	21.30	7.0	216.	-15.00	90.
7	10.00	8.00	2.34	8.00	13	36.	8.04	71.60	153.60	6.0	360.	-50.00	0.
8	4.00	4.00	4.00	4.00	1	36.	1.94	3.00	3.00	16.0	288.	40.00	0.
9	1.00	1.00	1.00	1.00	4	36.	0.79	0.20	0.20	1.0	360.	0.00	0.
10	4.00	3.50	4.00	3.50	2	36.	2.67	3.00	4.20	10.7	288.	20.00	0.
11	8.00	5.00	1.88	5.00	13	36.	5.26	17.6	65.2	5.9	360.	-25.00	0.
12	4.00	3.50	4.00	3.50	2	36.	2.67	3.00	4.20	10.7	288.	20.00	0.
13	8.00	5.00	1.88	5.00	13	36.	5.26	17.6	65.2	5.9	360.	-25.00	0.

EVALUATION OUTPUT - DETERMINATE TRUSS - FIRST DESIGN

DESIGN NO. 15

TRUSS MEMBER EVALUATION

	VR	AREQD	RREQD	AOPT	ROPT	WVE
MEMBER 1 TENSION, EXCEEDS (L/R)MAX	1.70		0.90			
MEMBER 1 OPTIMUM DESIGN				1.39	0.90	1.18
MEMBER 2 TENSION, EXCEEDS (L/R)MAX	1.70		0.90			
MEMBER 2 OPTIMUM DESIGN				1.39	0.90	1.18
MEMBER 3 TENSION, EXCEEDS (L/R)MAX	1.70		0.90			
MEMBER 3 OPTIMUM DESIGN				1.39	0.90	1.18
MEMBER 4 TENSION, EXCEEDS (L/R)MAX	1.70		0.90			
MEMBER 4 OPTIMUM DESIGN				0.69	0.90	2.36
MEMBER 5 COMPR., OVERSTRESSED	1.25	5.88				
MEMBER 5 OPTIMUM DESIGN				3.50	1.64	1.34
MEMBER 6 OPTIMUM DESIGN				2.47	1.38	1.90
MEMBER 7 OPTIMUM DESIGN				5.04	2.94	1.59
MEMBER 8 OPTIMUM DESIGN				1.85	1.20	1.05
MEMBER 9 ZERO STRESS						
MEMBER 10 TENSION, EXCEEDS (L/R)MAX	1.13		1.20			
MEMBER 10 OPTIMUM DESIGN				0.93	1.20	2.88
MEMBER 11 COMPR., OVERSTRESSED	1.24	6.50				
MEMBER 11 OPTIMUM DESIGN				4.27	2.26	1.23
MEMBER 12 TENSION, EXCEEDS (L/R)MAX	1.13		1.20			
MEMBER 12 OPTIMUM DESIGN				0.93	1.20	2.88
MEMBER 13 COMPR., OVERSTRESSED	1.24	6.50				
MEMBER 13 OPTIMUM DESIGN				4.27	2.26	1.23

SUMMARY FOR ENTIRE TRUSS

EVALUATION OF TRUSS WEIGHT

	TENSION	COMPRESSION	
TOTAL NO. OF MEMBERS EVALUATED			13
MEMBERS EXCEEDING (B/T)MAX		0	
MEMBERS EXCEEDING (L/R)MAX	6	0	
MEMBERS HAVING ZERO STRESS			1
OVERSTRESSED MEMBERS	0	3	
UNDERSTRESSED MEMBERS	7	2	
FULLY STRESSED MEMBERS	0	0	

TRUSS WEIGHT MERIT FACTOR = 1.508

EVALUATION OF JOINT COMPLEXITY

TOTAL NO. OF JOINTS EVALUATED	8
NO. OF JOINTS REQUIRING SINGLE GUSSET PLATES ONLY	2
NO. OF JOINTS REQUIRING DOUBLE GUSSET PLATES ONLY	0
NO. OF JOINTS REQUIRING MIXED CONNECTION	6

MEMBER WIDTH RATIOS FOR JOINTS WITH DOUBLE GUSSET PLATES

JOINT 1	1.00
JOINT 2	1.00
JOINT 5	1.00
JOINT 6	1.60
JOINT 7	1.00
JOINT 8	1.00

EVALUATION INPUT - DETERMINATE TRUSS - SECOND DESIGN

16	8	13	WELDED	
1	1	3	1	0.0
2	3	4	1	0.
3	4	5	1	0.0
4	5	2	1	0.0
5	6	7	11	90.0
6	7	8	3	90.0
7	1	6	11	0.0
8	3	6	1	0.0
9	3	7	4	0.0
10	4	7	1	0.0
11	7	5	13	0.0
12	5	8	1	0.0
13	8	2	13	0.0

1	3.00	3.00	3.00	3.00	1	36.	1.44	1.20	1.20	12.0	216.	30.00	0.
2	3.00	3.00	3.00	3.00	1	36.	1.44	1.20	1.20	12.0	216.	30.00	0.
3	3.00	3.00	3.00	3.00	1	36.	1.44	1.20	1.20	12.0	216.	30.00	0.
4	3.00	3.00	3.00	3.00	1	36.	1.09	0.96	0.96	16.0	216.	15.00	0.
5	6.20	6.02	6.20	6.02	11	36.	5.90	13.30	41.70	12.0	216.	-30.00	90.
6	3.97	6.50	3.97	6.50	3	36.	3.53	3.53	9.10	16.3	216.	-15.00	90.
7	9.94	7.99	9.94	7.99	11	36.	11.48	44.90	209.70	7.6	360.	-50.00	0.
8	4.00	4.00	4.00	4.00	1	36.	1.94	3.00	3.00	16.0	288.	40.00	0.
9	1.00	1.00	1.00	1.00	4	36.	0.79	0.20	0.20	1.0	360.	0.00	0.
10	4.00	4.00	4.00	4.00	1	36.	1.94	3.00	3.00	16.0	288.	20.00	0.
11	8.00	6.00	1.92	6.00	13	36.	4.78	26.00	96.10	5.6	360.	-25.00	0.
12	4.00	4.00	4.00	4.00	1	36.	1.94	3.00	3.00	16.0	288.	20.00	0.
13	8.00	6.00	1.92	6.00	13	36.	4.78	26.00	96.10	5.6	360.	-25.00	0.

EVALUATION OUTPUT - DETERMINATE TRUSS - SECOND DESIGN

DESIGN NO. 16

TRUSS MEMBER EVALUATION

	VR	AREQD	RREQD	AOPT	ROPT	WMF
MEMBER 1 OPTIMUM DESIGN				1.39	0.90	1.04
MEMBER 2 OPTIMUM DESIGN				1.39	0.90	1.04
MEMBER 3 OPTIMUM DESIGN				1.39	0.90	1.04
MEMBER 4 OPTIMUM DESIGN				0.69	0.90	1.57
MEMBER 5 OPTIMUM DESIGN				3.51	1.64	1.68
MEMBER 6 COMPR., EXCEEDS (L/R)MAX	1.08		1.08			
MEMBER 6 OPTIMUM DESIGN				2.76	1.31	1.28
MEMBER 7 OPTIMUM DESIGN				7.73	2.37	1.48
MEMBER 8 OPTIMUM DESIGN				1.85	1.20	1.05
MEMBER 9 ZERO STRESS						
MEMBER 10 OPTIMUM DESIGN				0.93	1.20	2.10
MEMBER 11 OPTIMUM DESIGN				3.81	2.39	1.26
MEMBER 12 OPTIMUM DESIGN				0.93	1.20	2.10
MEMBER 13 OPTIMUM DESIGN				3.81	2.39	1.26

SUMMARY FOR ENTIRE TRUSS

EVALUATION OF TRUSS WEIGHT

	TENSION	COMPRESSION	
TOTAL NO. OF MEMBERS EVALUATED			13
MEMBERS EXCEEDING (B/T)MAX		0	
MEMBERS EXCEEDING (L/R)MAX	0	1	
MEMBERS HAVING ZERO STRESS			1
OVERSTRESSED MEMBERS	0	0	
UNDERSTRESSED MEMBERS	7	4	
FULLY STRESSED MEMBERS	0	0	

TRUSS WEIGHT MERIT FACTOR = 1.416

EVALUATION OF JOINT COMPLEXITY

TOTAL NO. OF JOINTS EVALUATED	8
NO. OF JOINTS REQUIRING SINGLE GUSSET PLATES ONLY	3
NO. OF JOINTS REQUIRING DOUBLE GUSSET PLATES ONLY	0
NO. OF JOINTS REQUIRING MIXED CONNECTION	5

MEMBER WIDTH RATIOS FOR JOINTS HAVING DOUBLE GUSSET PLATES

JOINT 1	1.00
JOINT 2	1.20
JOINT 5	1.20
JOINT 6	1.60
JOINT 7	1.24
JOINT 8	1.20

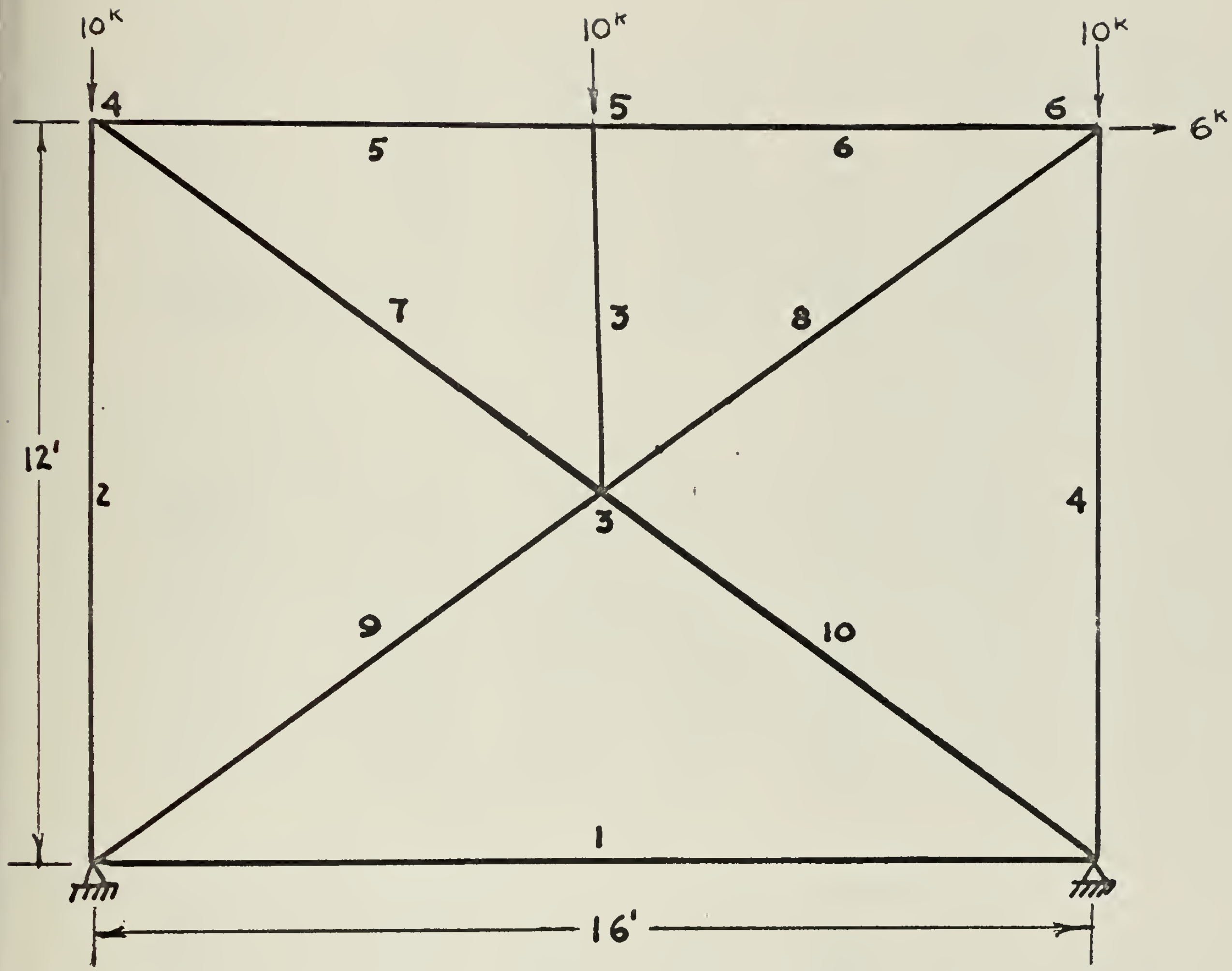


Fig. D-2 Statically indeterminate test truss.

	ANALYSIS		EVALUATION				
	A1	P, FROM STRESS	AOPT	ROPT	WMF	VIOL.	VR
CYCLE NO. 1							
MEMBER 1	3. 7	0.00	0.00	0.00	INF		
MEMBER 2	1.64	-8.92	1.48	0.91	1.11	L/R	1.36
MEMBER 3	1.64	-10.00	0.85	0.69	1.93		
MEMBER 4	1.64	-13.42	1.82	1.01	0.90	L/R	1.36
MEMBER 5	1.81	1.44	0.43	0.40	4.19		
MEMBER 6	1.81	1.44	0.43	0.40	4.19		
MEMBER 7	1.19	-1.81	2.99	0.60	0.40	L/R	1.46
MEMBER 8	1.64	5.69	0.45	0.50	3.67		
MEMBER 9	1.19	-2.64	2.99	0.60	0.40	L/R	1.46
MEMBER 10	1.19	-10.14	2.99	0.60	0.40	L/R	1.46
CYCLE NO. 2							
MEMBER 1	3. 7	0.00	0.00	0.00	INF		
MEMBER 2	1.48	-7.63	1.37	0.88	1.08		
MEMBER 3	0.85	-10.00	0.85	0.69	1.00		
MEMBER 4	1.82	-12.13	1.73	0.99	1.05		
MEMBER 5	0.43	3.16	0.17	0.40	2.50	L/R	1.07
MEMBER 6	0.43	3.16	0.17	0.40	2.50	L/R	1.07
MEMBER 7	2.99	-3.94	2.99	0.60	1.00		
MEMBER 8	0.45	3.55	0.19	0.50	2.33		
MEMBER 9	2.99	-4.78	2.99	0.60	1.00		
MEMBER 10	2.99	-12.28	3.15	0.61	0.95		
CYCLE NO. 3							
MEMBER 1	3.07	0.00	0.00	0.00	INF		
MEMBER 2	1.37	-7.76	1.38	0.88	0.99	P/A	1.02
MEMBER 3	0.85	-10.00	0.82	0.68	1.04		
MEMBER 4	1.73	-12.26	1.74	0.99	0.99	P/A	1.01
MEMBER 5	0.17	2.98	0.16	0.40	1.05		
MEMBER 6	0.17	2.98	0.16	0.40	1.05		
MEMBER 7	2.99	-3.72	2.99	0.60	1.00		
MEMBER 8	0.19	3.77	0.21	0.50	0.93	P/A	1.08
MEMBER 9	2.99	-4.56	2.99	0.60	1.00		
MEMBER 10	3.15	-12.05	3.12	0.61	1.01		

TAB. D-1. CONVERGENCE OF INDETERMINATE DESIGN CYCLE

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